Robust Control Tools for Validating UAS Flight Controllers

Slides prepared by **Micah Fry** (focusing on his contributions)
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Papers and Acknowledgements

- Following results drawn from the papers:
  - J. M. Fry and M. Farhood, “A comprehensive analytical tool for control validation of fixed-wing unmanned aircraft,” IEEE Transactions on Control Systems Technology, to appear.
  - M. Palframan, J. M. Fry, and M. Farhood, “Robustness analysis of flight controllers for fixed-wing unmanned aircraft systems using integral quadratic constraints,” IEEE Transactions on Control Systems Technology, Volume 27, Issue 1, Pages 86-102, January 2019.
  - J.M. Fry, M. Farhood, and P. Seiler, “IQC-based robustness analysis of discrete-time linear time-varying systems,” International Journal of Robust and Nonlinear Control, Volume 27, Issue 16, Pages 3135-3157, November 2017.

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  - NSF grant # CMMI-1351640
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Outline

- Motivation and approach
- Background
  - Notions in robust control theory
  - Integral quadratic constraint (IQC) theory
- Unmanned aircraft system (UAS) analysis framework
- Results and observations
- IQC theory for time-varying systems
- Future work

Certifying UAS Flight Controllers

- 26% of all DoD UAS mission failures are reportedly due to flight controller issues[1]
- Difficult to assess if a UAS controller will stabilize the aircraft and perform well
  - Claim can be made for *specific* cases
  - Cannot test/simulate ALL the configurations of the UAS
  - Certification methods ought to be faster and less expensive than standard techniques for manned aircraft
- Need a tool to quickly and inexpensively aid in certification of UAS flight controllers

[1]Office of the Secretary of Defense. (Dec. 2002). *Unmanned Aerial Vehicles Roadmap 2002-2027.*

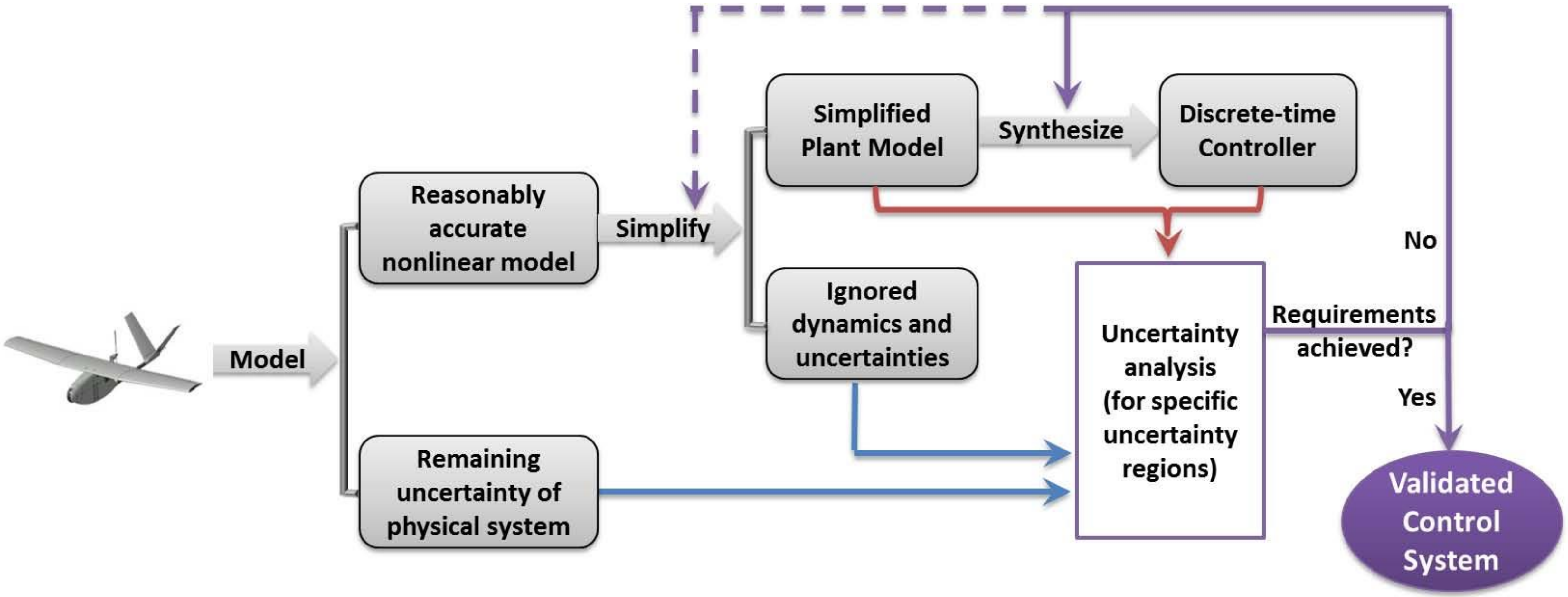
Photo Credit: C. Whitlock (2014, June 20). "When Drones Fall From the Sky" *The Washington Post*

Goals

- UAS dynamics are highly **nonlinear** and sensitive to **model uncertainties** and **external disturbances**

- Despite nonlinearities, uncertainties, and disturbances, we want to assert if a given control law will
  1. Stabilize the UAS
  2. Yield good performance
  3. Maintain safe behavior

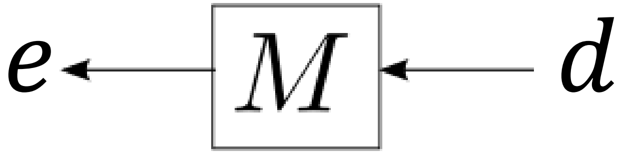
Algorithmic Level Validation



Robust Control

- M is a linear dynamic system

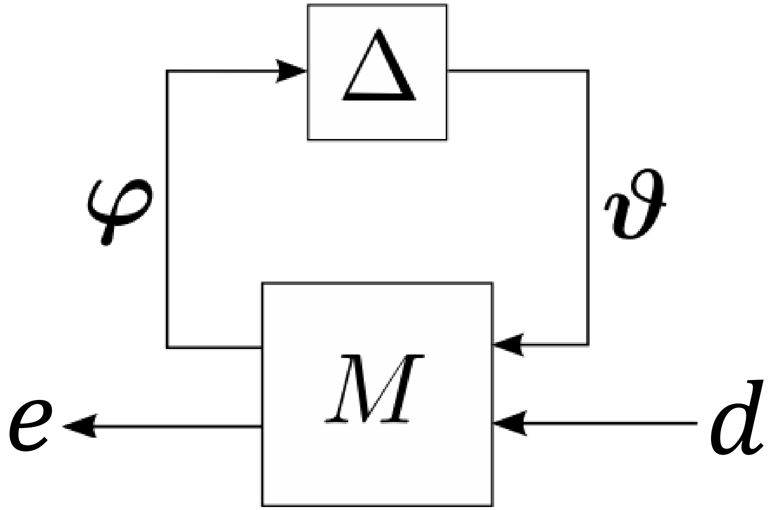
  $x(k+1) = Ax(k) + Bd(k), \quad e(k) = Cx(k) + Dd(k)$



- d is a disturbance signal (e.g. wind/noise)
- e is the performance output (e.g. position error)
- d belongs to the signal set $\mathcal{D} \subset \ell_2$
  - $\|d\|_{\ell_2}^2 = \sum_{k=0}^{\infty} d(k)^T d(k) < \infty$ (energy of signal d)
  - $\mathcal{D}$ is used to better characterize the disturbances
- The "size" of M is defined by the $\mathcal{D}$-to-ℓ_2-induced norm
  - $\|M\|_{\mathcal{D}\to\ell_2} = \sup_{0 \neq d \in \mathcal{D}} \frac{\|Md\|_{\ell_2}}{\|d\|_{\ell_2}}$
  - If $\mathcal{D} = \ell_2$, then the $\mathcal{D}$-to-ℓ_2-induced norm is the standard $\mathcal{H}_\infty$ norm

Robust Control

- Uncertainties are incorporated with the Δ block
  - $\Delta = \begin{bmatrix} \Delta_1 & & \\ & \Delta_2 & \\ & & \ddots \end{bmatrix} \in \boldsymbol{\Delta}$



- The interconnection (M, Δ) is an uncertain system
- Robust stability: $(I - M_{11}\Delta)^{-1}$ is well-defined, causal, and bounded on ℓ_2
- Robust $\mathcal{D}$-to-ℓ_2-gain performance level γ:
  - robustly stable + $\sup_{\Delta \in \boldsymbol{\Delta}} ||(M, \Delta)||_{\mathcal{D} \to \ell_2} \leq \gamma$

- Integral quadratic constraint (IQC) theory[2] provides such an upper bound γ

[2] A. Megretski and A. Rantzer, "System analysis via integral quadratic constraints," *IEEE Transactions on Automatic Control*, Volume 42, Issue 6, Pages 819-830, June 1997.

Integral Quadratic Constraints

- Expansive library expressing different uncertainty groups (nonlinearities, time-varying, dynamic, etc.)

- Allows limiting disturbances to a specified signal set $\mathcal{D} \subset \ell_2$

- Unifying approach

- Provides sufficient condition expressed as a linear matrix inequality

$$F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n \preccurlyeq 0$$

Integral Quadratic Constraints

- An uncertainty Δ satisfies the IQC defined by $\Pi(e^{j\omega}) = \Pi(e^{j\omega})^* \in \mathcal{RL}_\infty$ if
  - $\begin{bmatrix} I \\ \Delta \end{bmatrix}^* \Pi \begin{bmatrix} I \\ \Delta \end{bmatrix} \succcurlyeq 0$ (denoted by $\Delta \in \mathrm{IQC}(\Pi)$)
- A signal set $\mathcal{D} \subset \ell_2$ satisfies the signal IQC defined by $\Phi(e^{j\omega}) = \Phi(e^{j\omega})^*$ if
  - $\langle d, \Phi d \rangle_{\ell_2} \geq 0$ for all $d \in \mathcal{D}$ (denoted by $\mathcal{D} \in \mathrm{SigIQC}(\Phi)$)
- Given an <u>IQC multiplier</u> Π, a <u>signal IQC multiplier</u> Φ and performance level γ:
  - Define the augmented IQC multiplier $\tilde{\Pi} = \begin{bmatrix} \Pi_{11} & 0 & \Pi_{12} & 0 \\ 0 & I & 0 & 0 \\ \Pi_{12}^* & 0 & \Pi_{22} & 0 \\ 0 & 0 & 0 & \Phi - \gamma^2 I \end{bmatrix}$

Integral Quadratic Constraints

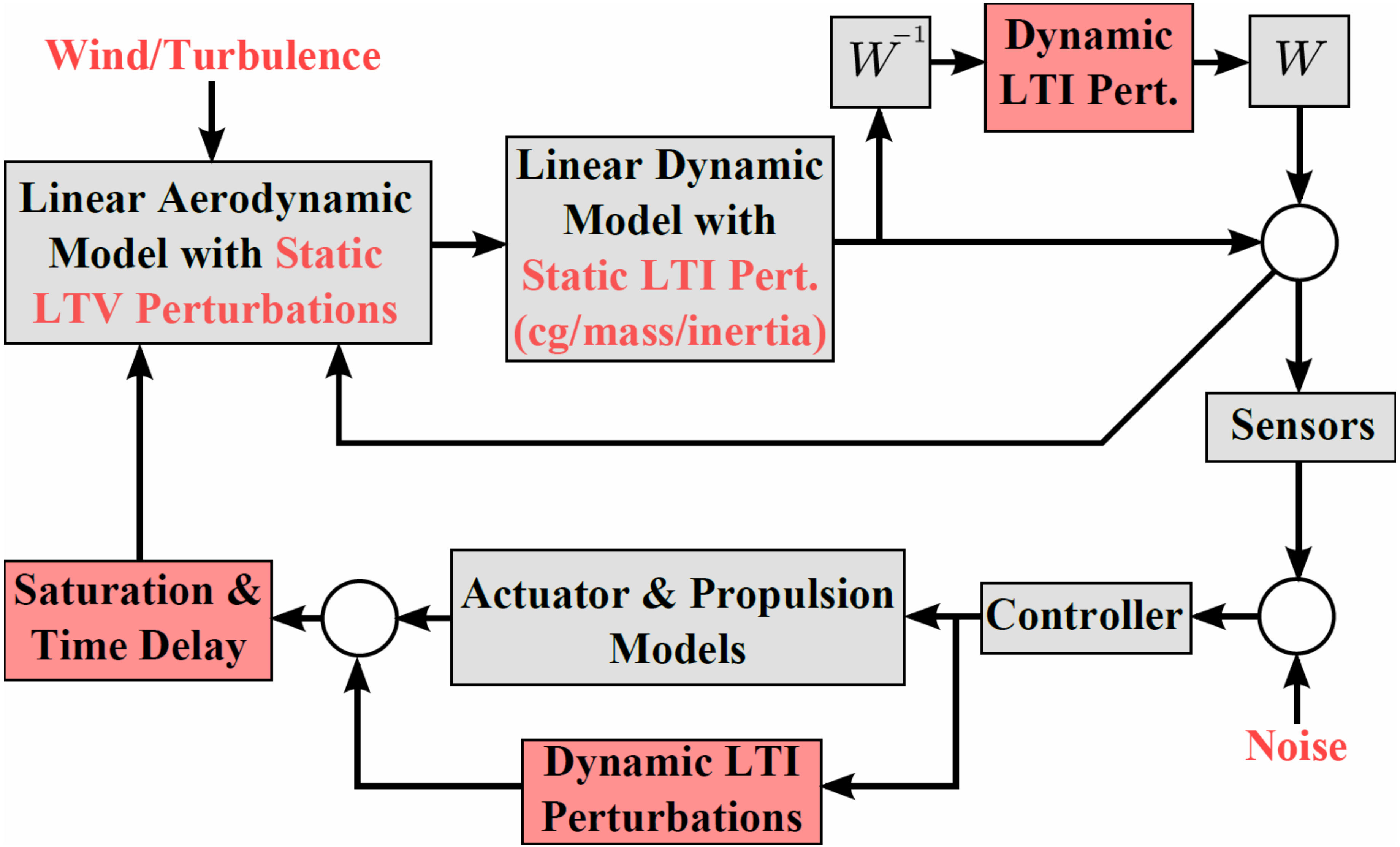
- IQC Theorem:
  - Given an interconnection (M, Δ), if for all $\tau \in [0,1]$:
    - $(I - \tau M_{11}\Delta)^{-1}$ is well-defined and causal
    - $\tau\Delta \in \mathrm{IQC}(\Pi)$
    - $\mathcal{D} \in \mathrm{SigIQC}(\Phi)$
    - $\begin{bmatrix} M \\ I \end{bmatrix}^* \tilde{\Pi} \begin{bmatrix} M \\ I \end{bmatrix} \preccurlyeq -\epsilon I$ (where $\epsilon > 0$)
  - Then:
    - (M, Δ) has a robust $\mathcal{D}$-to-ℓ_2-gain performance level of γ

IQC analysis framework for UAS

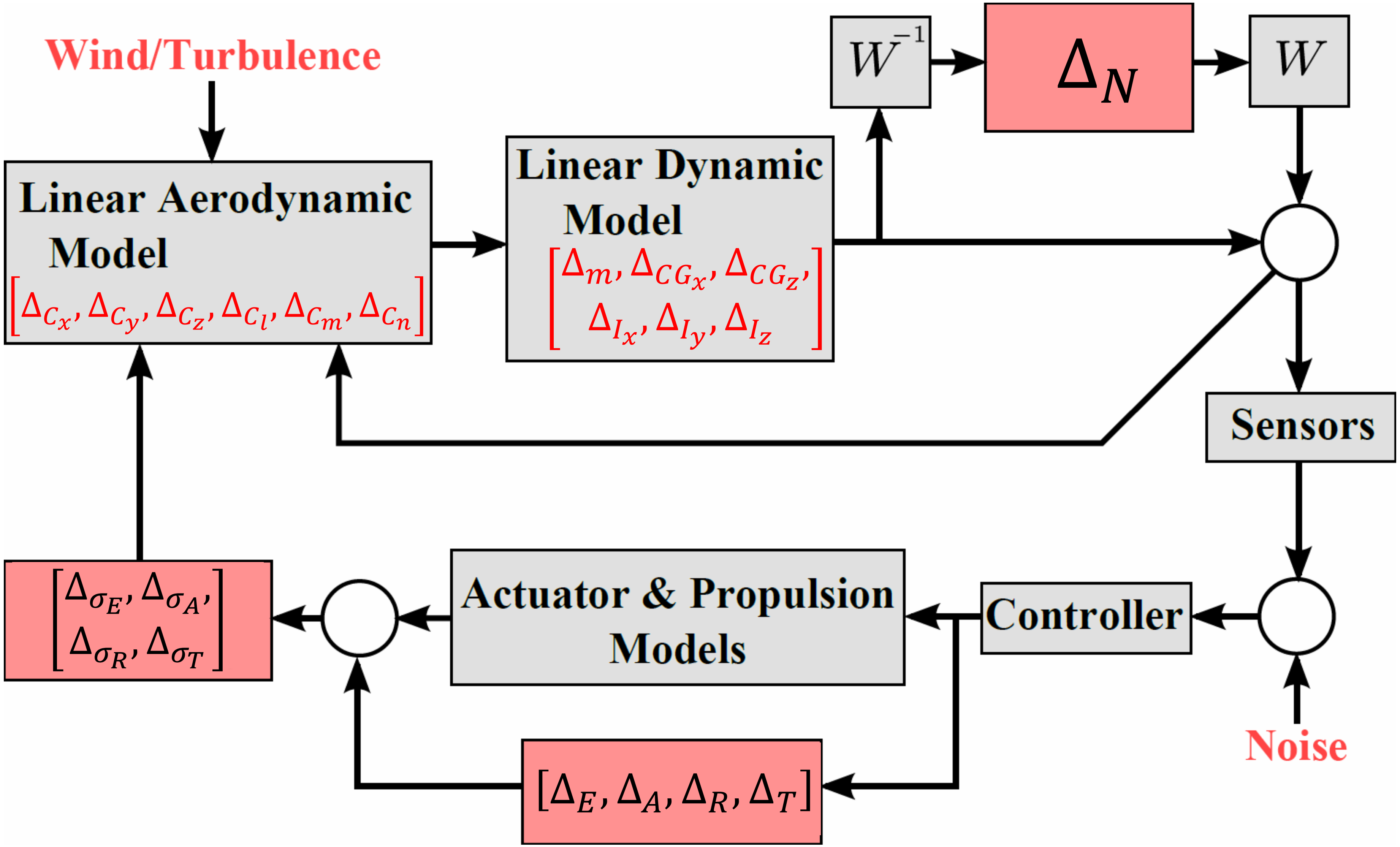
- Work has been done in deriving a framework for analysis of uncertain UAS[3]
  - Uncertainties inherent to the UAS are characterized and quantified
  - IQC analysis is conducted to identify sensitivities and compare controllers
  - Signal IQCs are utilized to significantly reduce conservativeness of analysis results
  - A controller tuning routine using IQC analysis is developed
  - Framework is validated by conducting flight tests

[3] M. Palframan, J. M. Fry, and M. Farhood, "Robustness analysis of flight controllers for fixed-wing unmanned aircraft systems using integral quadratic constraints," IEEE Transactions on Control Systems Technology, Volume 27, Issue 1, Pages 86-102, January 2019.

UAS IQC Framework Overview



UAS IQC Framework Overview



UAS IQC Framework Overview

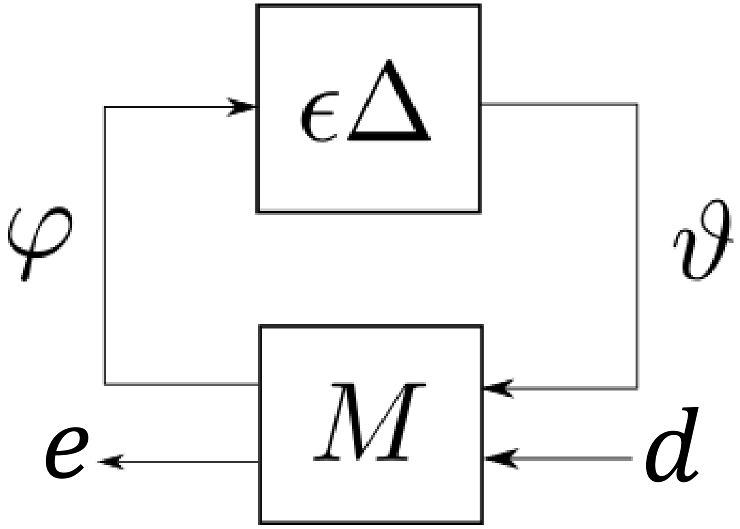
Uncertainty	Type	Bounds
Δ_{C_x}	1 x 1 RB-SLTV	$-0.054 \leq \Delta_{C_x}(k) \leq 0.026$
Δ_{C_y}	1 x 1 RB-SLTV	$-0.045 \leq \Delta_{C_y} \leq 0.037$
Δ_{C_z}	1 x 1 RB-SLTV	$-0.113 \leq \Delta_{C_z} \leq 0.119$
Δ_{C_l}	1 x 1 RB-SLTV	$-0.022 \leq \Delta_{C_l} \leq 0.026$
Δ_{C_m}	1 x 1 RB-SLTV	$-0.120 \leq \Delta_{C_m} \leq 0.125$
Δ_{C_n}	1 x 1 RB-SLTV	$-0.006 \leq \Delta_{C_n} \leq 0.006$
Δ_E	1 x 1 DLTI	$\|\Delta_E\|_\infty \leq 0.05$
Δ_A	1 x 1 DLTI	$\|\Delta_A\|_\infty \leq 0.05$
Δ_R	1 x 1 DLTI	$\|\Delta_R\|_\infty \leq 0.05$
Δ_T	1 x 1 DLTI	$\|\Delta_T\|_\infty \leq 0.2$

Uncertainty	Type	Bounds
Δ_{σ_E}	1 x 1 RB-SLTV	$0 \leq \Delta_{\sigma_E} \leq 0.1$
Δ_{σ_A}	1 x 1 RB-SLTV	$0 \leq \Delta_{\sigma_A} \leq 0.1$
Δ_{σ_R}	1 x 1 RB-SLTV	$0 \leq \Delta_{\sigma_R} \leq 0.1$
Δ_{σ_T}	1 x 1 RB-SLTV	$0 \leq \Delta_{\sigma_T} \leq 0.1$
Δ_N	12 x 12 DLTI	$\|\Delta_N\|_\infty \leq 0.01$
Δ_m	6 x 6 SLTI	$-0.57 \leq \Delta_m \leq 0.57$
Δ_{CG_x}	4 x 4 SLTI	$0 \leq \Delta_{CG_x} \leq 0.03$
Δ_{CG_z}	3 x 3 SLTI	$-0.03 \leq \Delta_{CG_z} \leq 0.03$
Δ_{I_x}	1 x 1 SLTI	$-0.20 \leq \Delta_{I_x} \leq 0.20$
Δ_{I_y}	3 x 3 SLTI	$-0.23 \leq \Delta_{I_y} \leq 0.23$
Δ_{I_z}	1 x 1 SLTI	$-0.28 \leq \Delta_{I_z} \leq 0.28$

- Signal IQC multipliers are also used to characterize sensor noise

Results

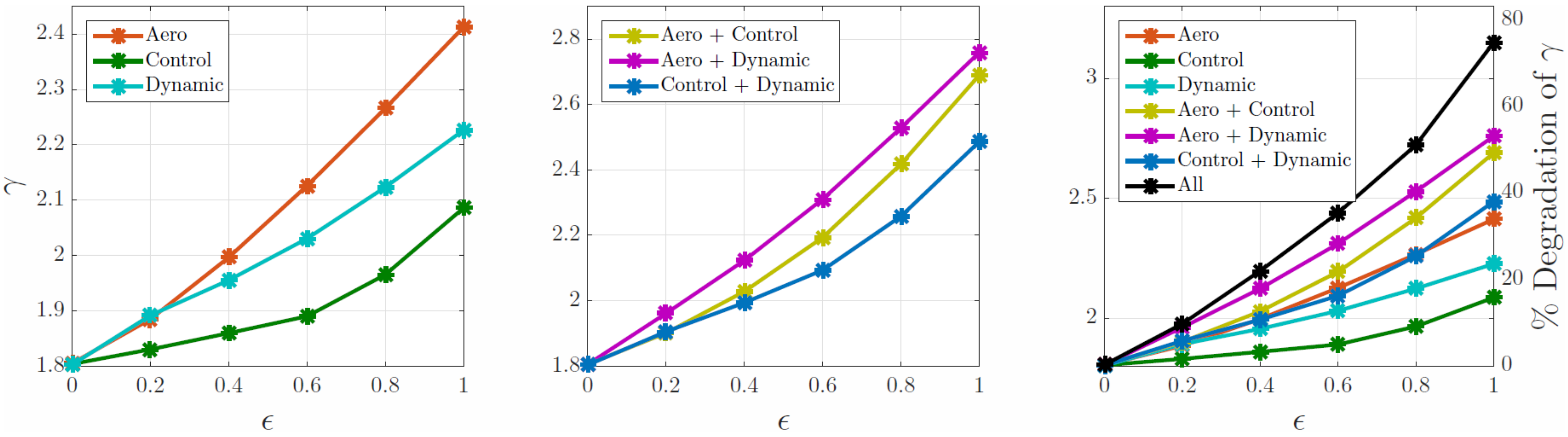
- Using MATLAB, the previous framework produces uncertain UAS model
- Uncertainties are scaled with $\epsilon \in [0,1]$



- IQC analysis is conducted by solving a semi-definite program

Results (Sensitivities)

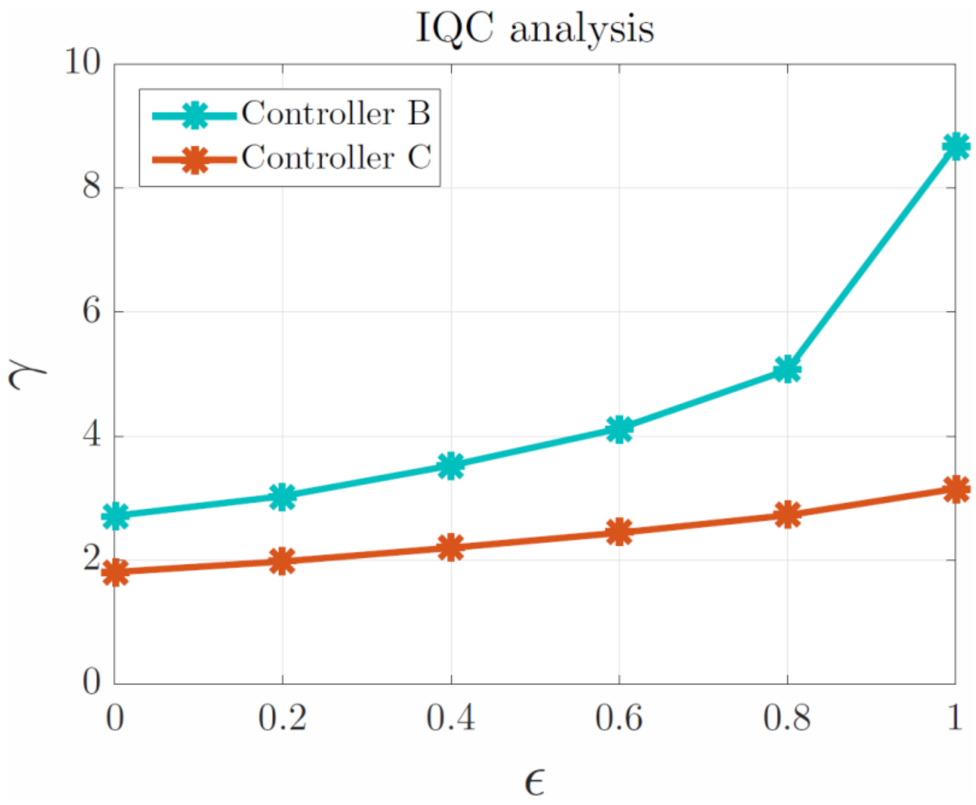
- Given a controller, IQC analysis is conducted on uncertain UAS



- Analysis done on separate and combined groups
- Reveals sensitivities to uncertainties
- % Degradation of performance increases nonlinearly

Results (Comparison)

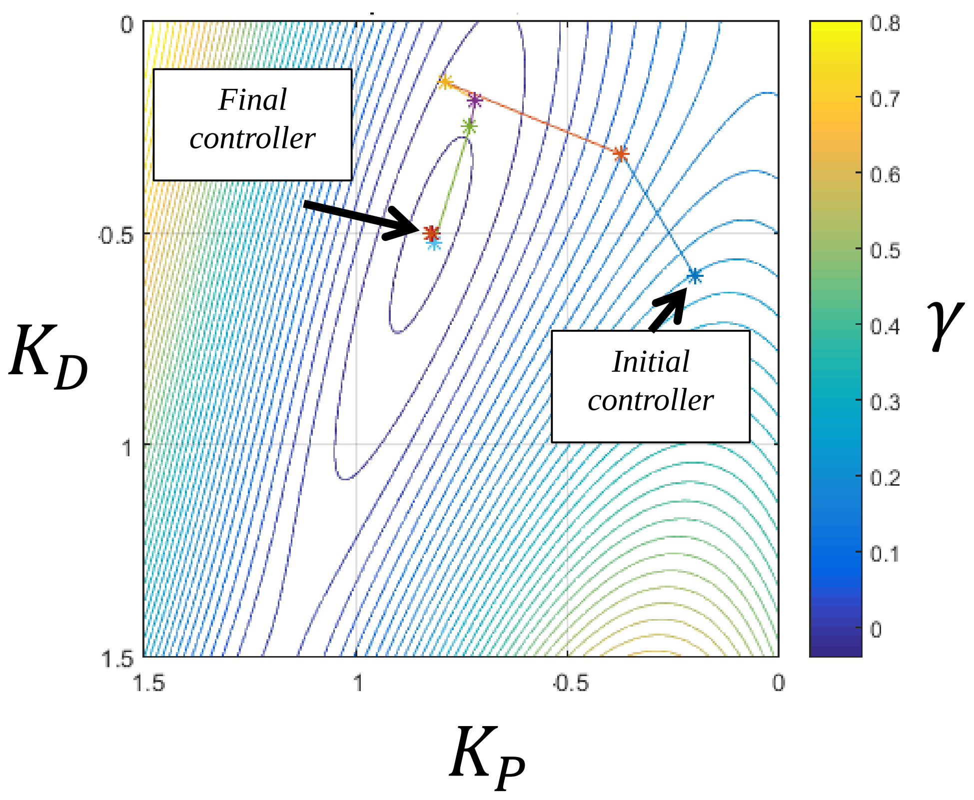
- Comparing one controller against another



- Demonstrates improved γ-value AND reduced sensitivity to uncertainties

Results (Tuning)

- Vary controller design parameters to iteratively find controller which yields improved γ-values
- Example controller design parameters:
  - PID: $\boldsymbol{K_P, K_I, K_D}$
  - LQR: $\boldsymbol{Q}$ and $\boldsymbol{R}$ matrices
- Implement BFGS algorithm for solving nonlinear optimization problem

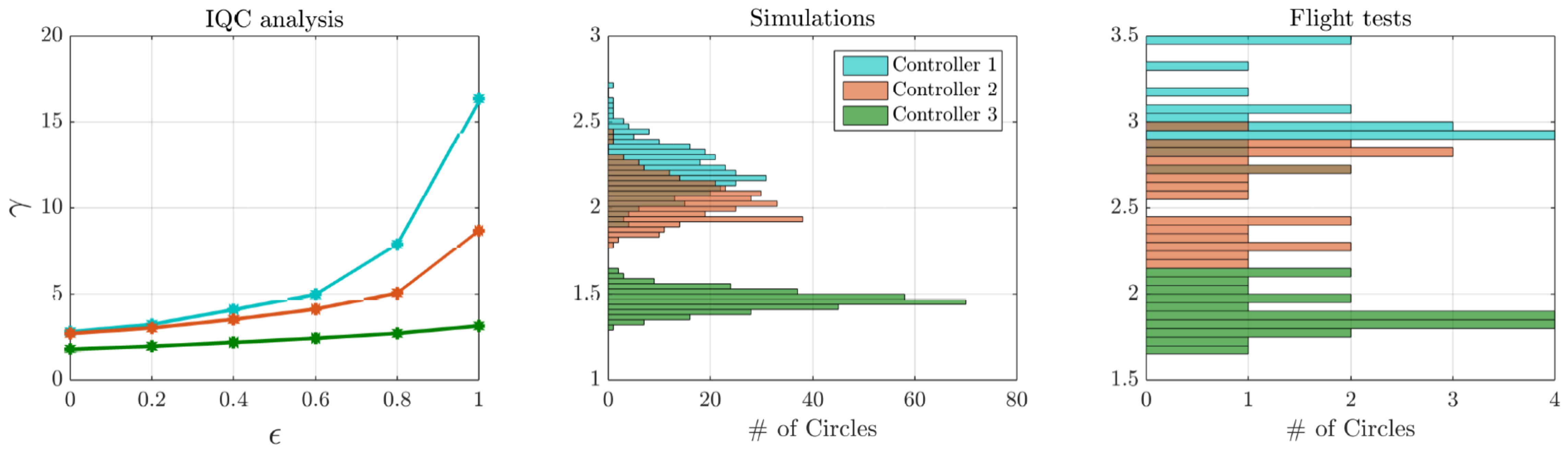


Validating Framework

- Validation process for uncertain UAS framework:
  - Tune a controller using IQC analysis
  - Conduct IQC analysis on initial, intermediate, and final controller
  - Conduct Monte-Carlo simulations of uncertain UAS with controllers
  - Conduct flight tests with controllers
  - Compare γ-values obtained from IQC analysis, simulations, and flight tests
  - https://www.youtube.com/watch?v=2HvmhOieRS0&t=17s

Results (Validation)

- Tuning routine starts with bad controller, ends with good controller



- Confirms that IQC analysis can qualitatively compare and tune controllers
- Simulations are over-optimistic while IQC analysis is conservative

Needed Work

- The previous results applied for
  - a single controller type (trajectory-tracking H_∞)
  - flying a single maneuver (level circle)

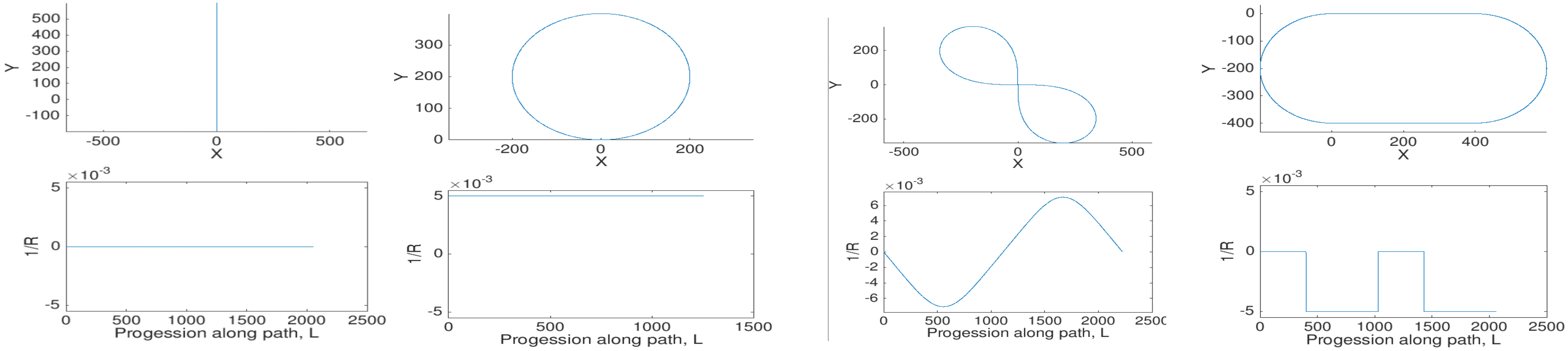
- How can we apply IQC analysis to a suite of maneuvers?

- How well does IQC analysis predict performance for different controller types[4]?

[4] J. M. Fry and M. Farhood, “A comprehensive analytical tool for control validation of fixed-wing unmanned aircraft,” IEEE Transactions on Control Systems Technology, to appear.

Sets of Admissible Paths

- A level path may be characterized by the history of its *radius of curvature (R)*



Straight line *Circle* *Figure-8* *Racetrack*

- The effect of *R* may be incorporated in the UAS dynamics as an uncertainty
- Enforcing $|R| \geq r_c$ signifies that IQC analysis applies for executing any level path with a bounded radius of curvature

Different UAS Controllers

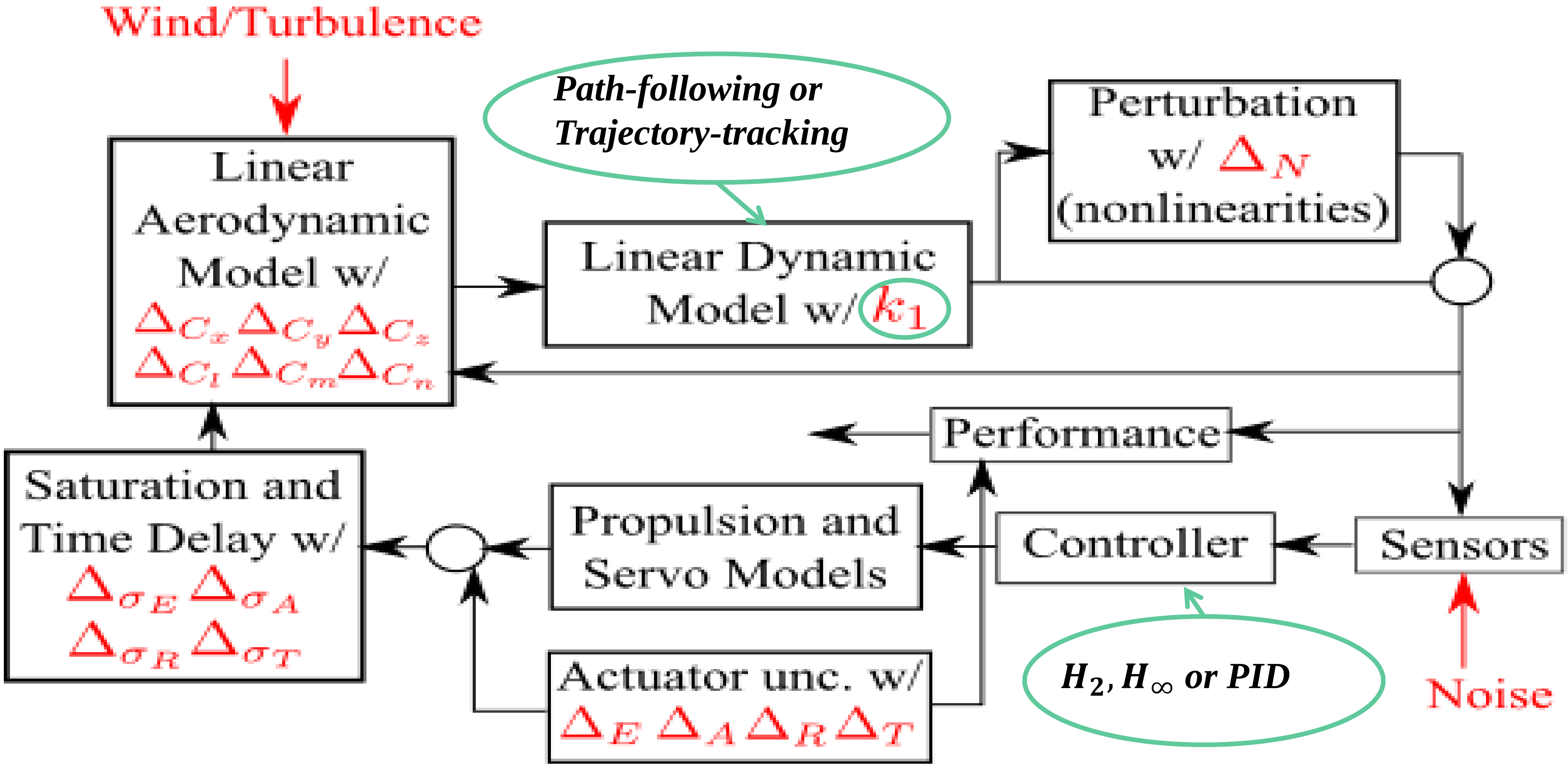
- Given a path, UAS control is approached in two ways:
  - Path-following (stay on a 3D path)
  - Trajectory-tracking (be at a certain place at a certain time)

- Most off-the-shelf UAS controllers are path-following

- Building off previous work[5], new UAS path-following dynamics are expressed

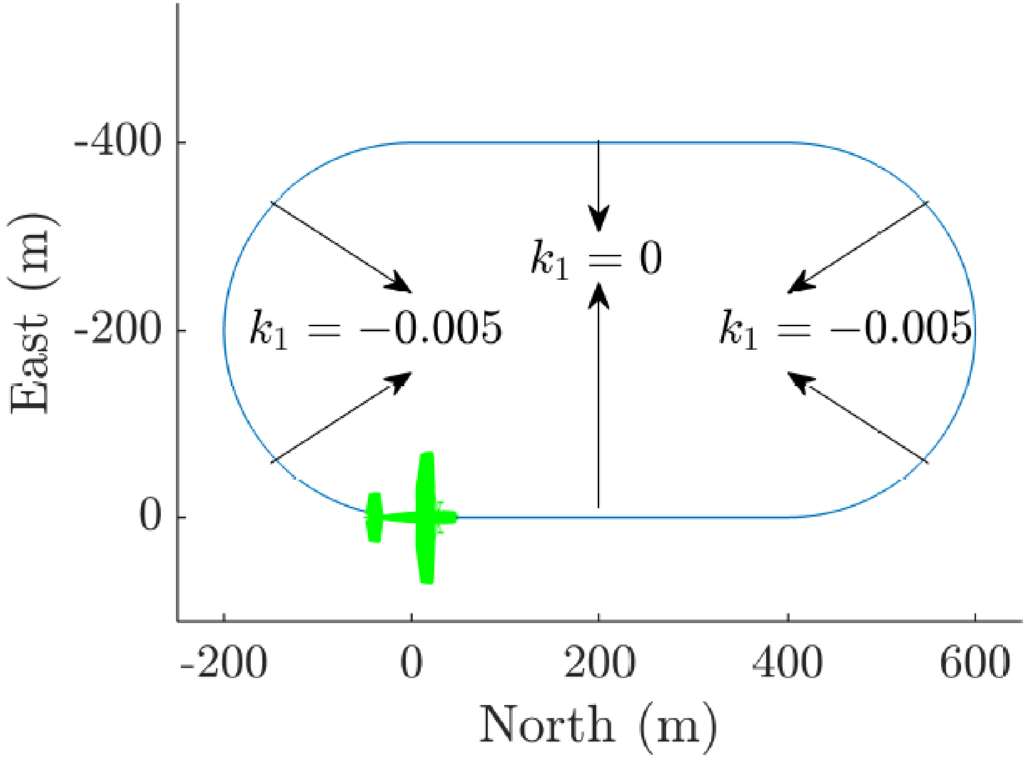
[5] I. Kaminer, A. Pascoal, E. Xargay, N. Hovakimyan, C. Cao, and V. Dobrokhodov, "Path following for small unmanned aerial vehicles using L1 adaptive augmentation of commercial autopilots," *Journal of Guidance, Control, and Dynamics*, Volume 33, Issue 2, Pages 550-564, 2010.

Updated framework



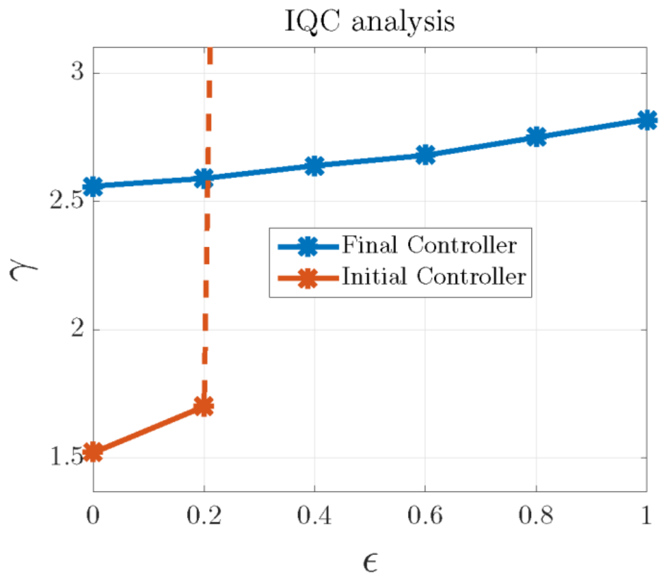
Validation Procedure

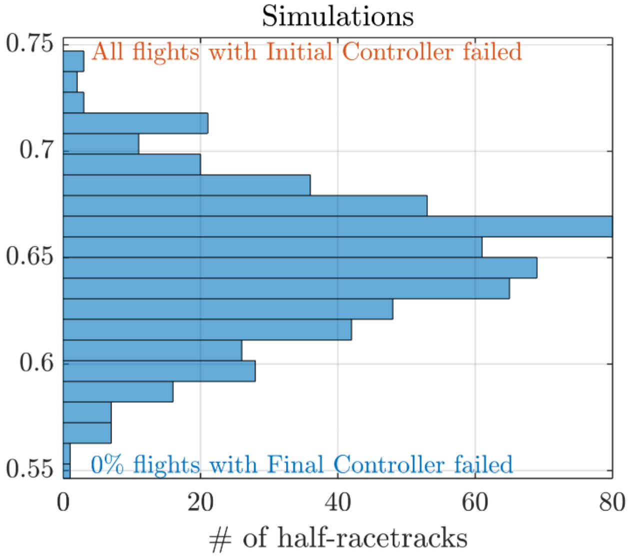
- Repeat previous validation procedure (analysis, simulations, flight tests)
- Five controller types
  - Trajectory-tracking H_∞
  - Trajectory-tracking H_2
  - Path-following H_∞
  - Path-following H_2
  - Path-following PID
- Executing a racetrack maneuver

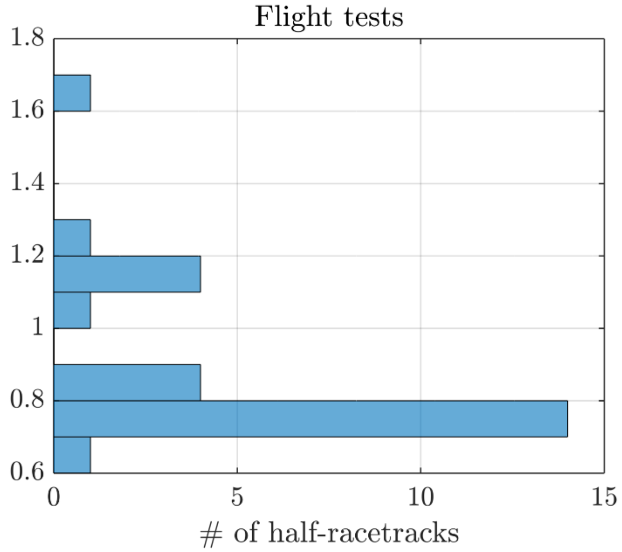


- https://www.youtube.com/watch?v=pSwoEPrc56k&t=217s

Results (Path-following H_∞)





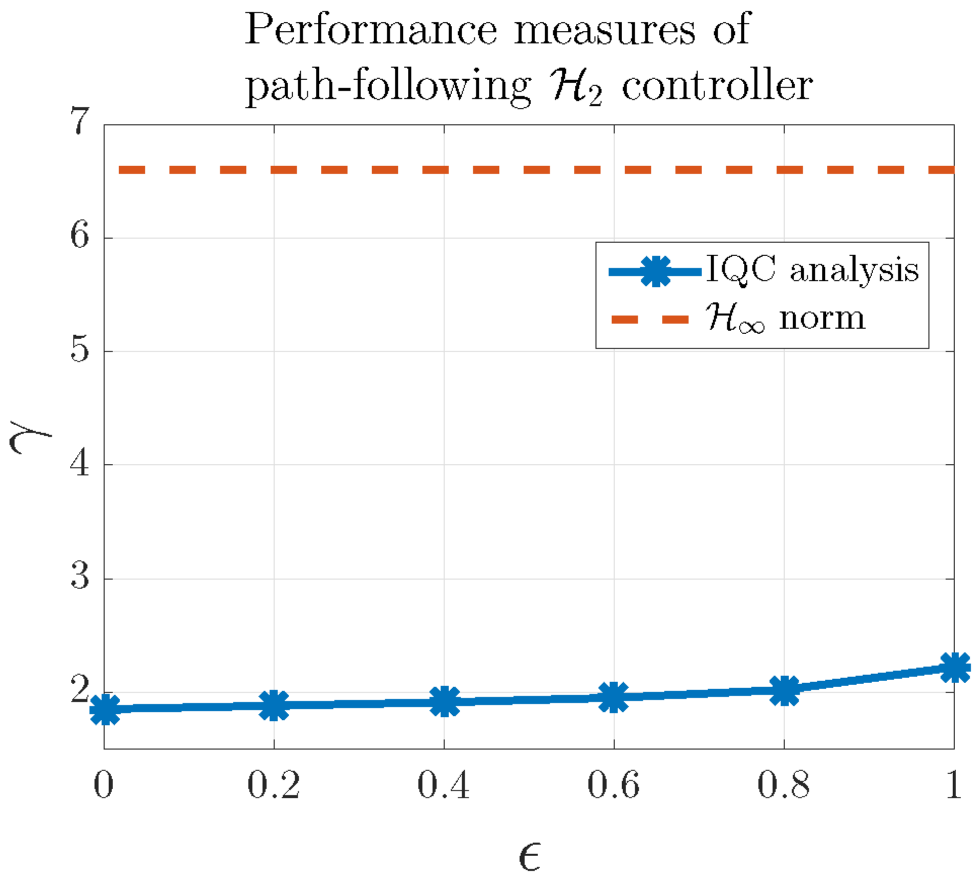


$\gamma_{final}^{IQC} = 2.82$ $\max(\gamma_{final}^{SIM}) = 0.74$ $\max(\gamma_{final}^{TEST}) = 1.64$

- Final controller is ***robust*** to uncertainties
- IQC analysis predicts initial controller performs better than final controller ***without*** uncertainties
- Initial controller with uncertainties will ***fail***; confirmed by simulation

Interesting Observation 1

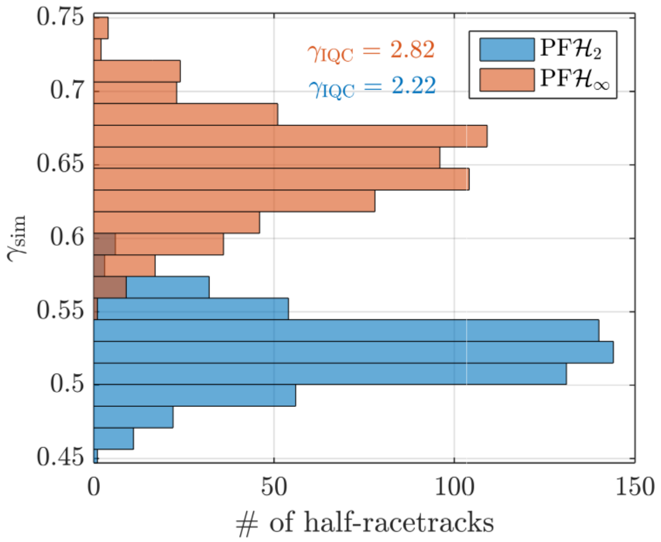
- Providing information on the disturbances (wind/sensor noise) is VERY helpful
- γ-value for the uncertain system is less than the $\mathcal{H}_\infty$ norm of the nominal system
- This is because IQC analysis is restricted to appropriate types of disturbances!
  - Wind = constant + Dryden model turbulence
  - Sensor noise = white noise signals



Interesting Observation 2

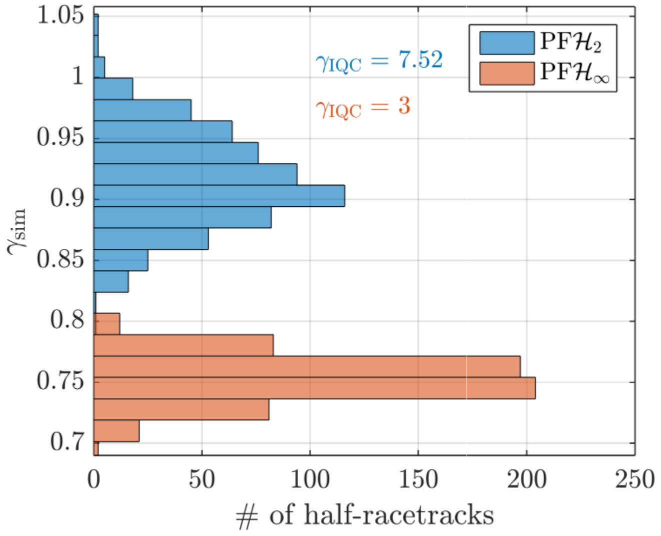
- Not only can signal IQCs reduce conservatism, they help make improved predictions
- These simulations are conducted by assuming wind consists of constant wind + turbulence

  (γ_{IQC} obtained with pertinent signal IQCs)



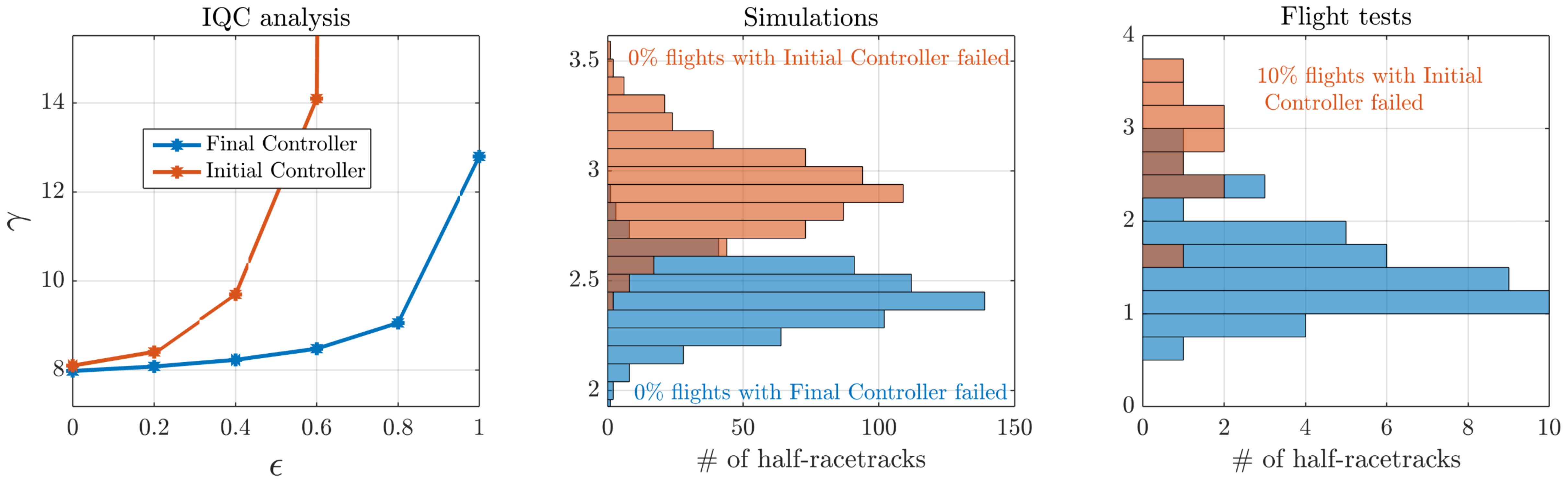
- If simulations allows wind to be more general (being more conservative) these comparisons flip

  (γ_{IQC} obtained w/o signal IQCs, i.e., $\mathcal{D} = \ell_2$)



Interesting Observation 3

- A previous PID controller provided an interesting case study



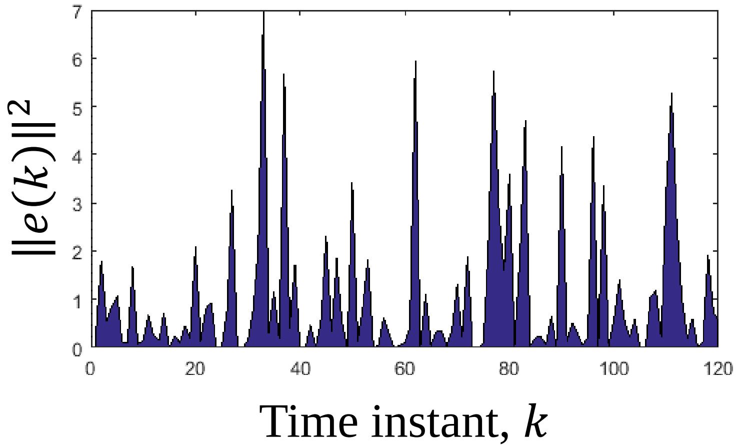
- IQC analysis concluded the initial controller **was not** robust
- Simulations predicted the initial controller **was** robust
- During flight tests, the initial controller **failed**

Recap Results

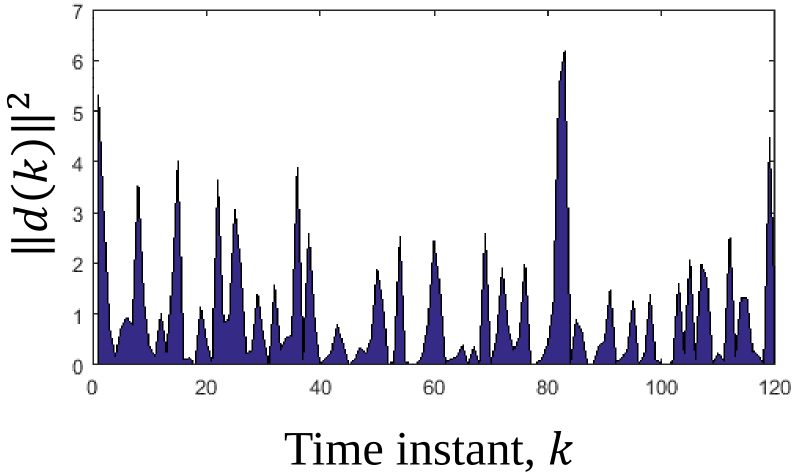
- The previous results demonstrate:
  - How to derive the uncertain UAS model
  - How IQC analysis generates γ-values to
    - Identify sensitivities to uncertainties
    - Compare controllers
    - Tune controllers
    - Predict loss-of-control where simulation may not
- Can IQC analysis be used to bound the UAS states?

Interlude on γ-values

- Mathematical meaning of γ:
  - For any disturbance $d \in \mathcal{D}$ and any uncertainty $\Delta \in \boldsymbol{\Delta}$
    - **energy of the performance signal** will be less than or equal to the **energy of the disturbance signal** scaled by γ^2
    - $\|e\|_{\ell_2} \leq \gamma \|d\|_{\ell_2}$ for all $\Delta \in \boldsymbol{\Delta}, d \in \mathcal{D}$



$$\|e\|_{\ell_2}^2 \leq \gamma^2 \|d\|_{\ell_2}^2$$



- Previous results demonstrated γ is a useful metric, but it isn't too intuitive
- If $e(k) = 0$ at every time *except* a single instance $\tilde{k}$, we could bound the output at $\tilde{k}$

IQC Theory for Time-Varying Systems

- Recall that the IQC theorem is concerned with asserting the inequality:
  - $\begin{bmatrix} M \\ I \end{bmatrix}^* \tilde{\Pi} \begin{bmatrix} M \\ I \end{bmatrix} \preccurlyeq -\epsilon I$
- If the system M and augmented multiplier Π are LTI, this operator inequality becomes
  - $\begin{bmatrix} M(e^{j\omega}) \\ I \end{bmatrix}^* \tilde{\Pi}(e^{j\omega}) \begin{bmatrix} M(e^{j\omega}) \\ I \end{bmatrix} \preccurlyeq -\epsilon I$ for all $\omega \in \mathbb{R} \cup \{\infty\}$
- Via the KYP lemma, this frequency-domain inequality (having infinite constraints) is equivalent to the existence of a $P = P^T$ and such that
  - $\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}^T \begin{bmatrix} -P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & \tilde{S} \end{bmatrix} \begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} \preccurlyeq -\epsilon I$, where $\tilde{\Pi} = \tilde{\Psi}^* \tilde{S} \tilde{\Psi}$ & $[\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}]$ is a realization of $\tilde{\Psi} \begin{bmatrix} M \\ I \end{bmatrix}$
- What if M and/or Π are time-varying? We get stuck at the operator inequality

IQC Theory for Time-Varying Systems

- We have proven that a set of similar LMIs provide similar robustness guarantees
  - M and Π can be time varying
- IQC Theorem[6]:
  - Given an interconnection (M, Δ), if:
    - $(I - M_{11}\Delta)^{-1}$ is well-defined and causal
    - $\Delta \in \mathrm{IQC}(\Pi)$ and $\Pi_{11} \succcurlyeq \beta I, \quad \Pi_{22} \preccurlyeq -\beta I$ (where $\beta > 0$)
    - There exist a sequence $P(k) = P(k)^T$ and scalar $\epsilon > 0$ such that:

$$\begin{bmatrix} I & 0 \\ \tilde{A}(k) & \tilde{B}(k) \\ \tilde{C}(k) & \tilde{D}(k) \end{bmatrix}^T \begin{bmatrix} -P(k) & 0 & 0 \\ 0 & P(k+1) & 0 \\ 0 & 0 & \tilde{S}(k) \end{bmatrix} \begin{bmatrix} I & 0 \\ \tilde{A}(k) & \tilde{B}(k) \\ \tilde{C}(k) & \tilde{D}(k) \end{bmatrix} \preccurlyeq -\epsilon I$$

  - Then:
    - (M, Δ) has a robust ℓ_2-gain performance level of γ

[6] J.M. Fry, M. Farhood, and P. Seiler, "IQC-based robustness analysis of discrete-time linear time-varying systems," International Journal of Robust and Nonlinear Control, Volume 27, Issue 16, Pages 3135-3157, November 2017.

IQC Theory for Time-Varying Systems

- Definition: A sequence of matrices $Q(k)$ is (h,q)-eventually periodic if
  - $Q(h+q+k) = Q(h+k)$ for all $k \in \{0,1,2,\ldots\}$ ($h \in \{0,1,\ldots\}, q \in \{1,2,\ldots\}$)
- Definition: An LTV system M is (h,q)-eventually periodic if
  - The state-space matrices $A(k), B(k), C(k)$, and $D(k)$ are (h,q)-eventually periodic
- Corollary:
  - If M is an (h_M, q_M)-eventually periodic system

    Π is an (h_Π, q_Π)-eventually periodic system

    and defining h as $\max(h_M, h_\Pi)$ and q as the least common multiple of q_M and q_Π
  - Then the existence of a general sequence $P(k) = P(k)^T$ satisfying the previous LMIs is equivalent to the existence of an (h,q)-eventually periodic sequence $P_{h,q}(k)$ satisfying the previous LMIs
- This result enables application of computationally tractable semidefinite programs

Implications of IQC theorem

- Given a system M with state-space matrix sequences $A(k), B(k), C(k)$, and $D(k)$, construct a finite horizon system $\bar{M}_h$ of horizon length h as follows:

$0 \le k < h-1$:

$$\bar{A}(k) = A(k), \qquad \bar{B}(k) = B(k),$$
$$\bar{C}_1(k) = C_1(k), \qquad \bar{D}_{1i}(k) = D_{1i}(k), i = 1,2$$
$$\bar{C}_2(k) = 0, \qquad \bar{D}_{2i}(k) = 0,$$

$k = h-1$:

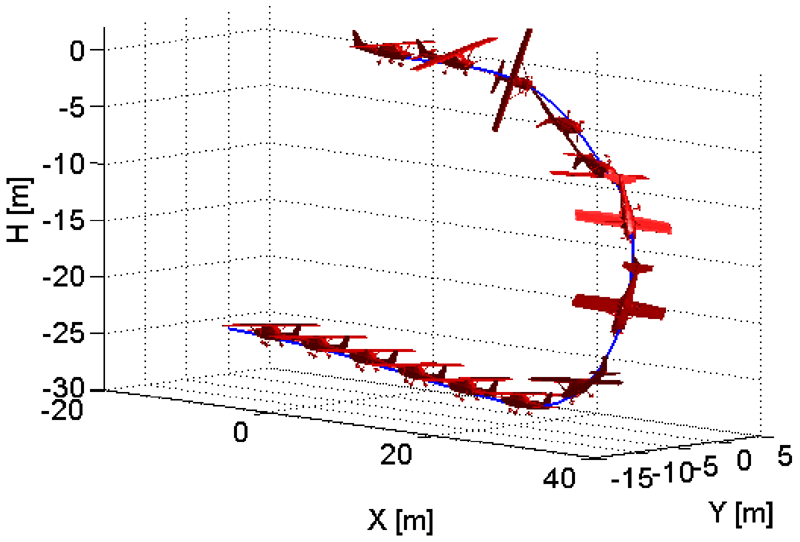
$$\bar{A}(k) = A(k), \qquad \bar{B}(k) = B(k),$$
$$\bar{C}_1(k) = C_1(k), \qquad \bar{D}_{1i}(k) = D_{1i}(k),$$
$$\bar{C}_2(k) = C_2(k), \qquad \bar{D}_{2i}(k) = D_{2i}(k),$$

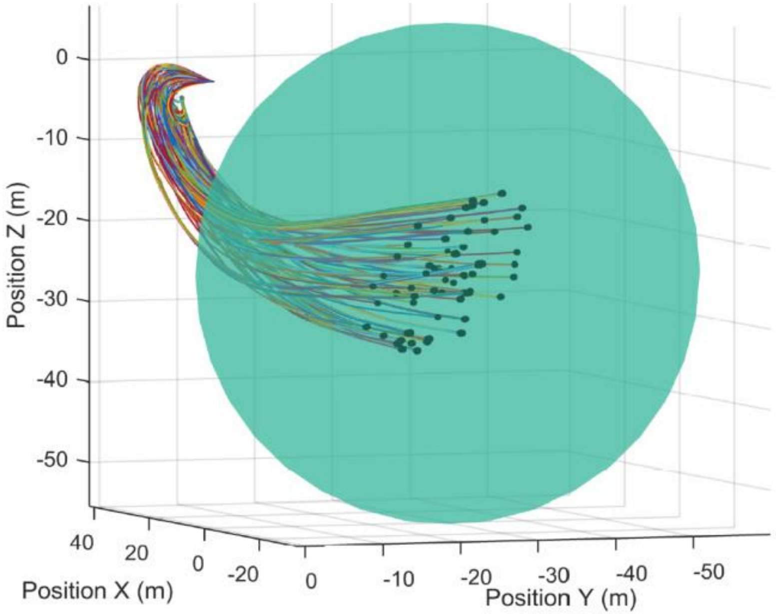
$k \ge h$: All matrices set to zero

- $\bar{M}_h$ is an $(h, 1)$-eventually periodic system
- IQC analysis provides: $\|e(h-1)\|_{\mathbb{R}^n} \le \gamma \|d\|_{\ell_2}$

Worst-case bounds

- γ-value may now be used to define bounding ellipsoids at each time instant
- Example:
  - Analysis of position of uncertain UAS at the end of a Split-S maneuver
  - Assumption that aircraft begins at known initial condition





Future work

- Incorporate uncertain initial conditions
- Reduce conservatism