Simulation of Multi-Mode DAEs Systems

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FEANICSES Workshop

ISAE-Supaero, Toulouse, France

June 20th, 2019

Define/Describe **Reactions** to **Events**

Events

Model a simplified (often discrete) perception of the rich environment.

Reactions

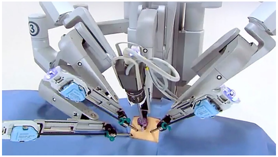
Model how the system is supposed to react to an event so that it respects a set of constraints which are essentially physics laws and/or predefined requirements.

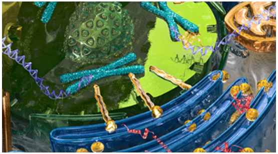
```
Init
⟶
[
(
Sensing:  read data from sensors
Control:  actuate
Plant:  evolve ◀ ◀ ◀ ◀ ◀
)*
]
Safety
```

Evolution

- Continuous time
- Differential Equations (ODE, DAE, etc.)

Convenient Model for a Large Class of Systems





Challenges

Invariants generation

- Reachable, maximal positively invariant sets, etc.
- **Approximate** exact computations to scale

Simulation of multi-mode DAE

- Well-founded operational semantics (compilation, index reduction)
- Preserve composionality in presence of mode changes
- Proper handling of zero-crossing
  (detection and consistent initialization)
- Cascades of zero-crossings (sliding modes)

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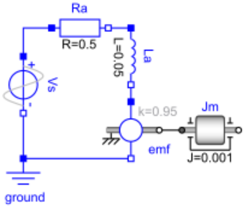
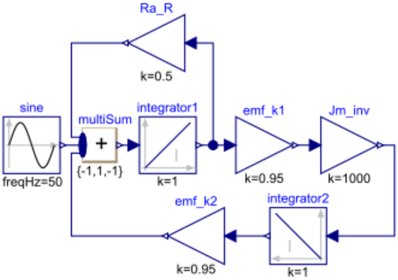
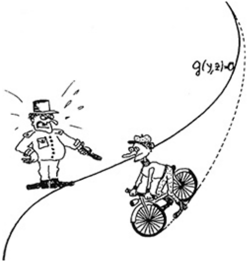

Ra
R=0.5
L=0.05
La
Vs
k=0.95
Jm
emf
J=0.001
ground


Ra_R
k=0.5
sine
multiSum
integrator1
emf_k1
Jm_inv
freqHz=50
{-1,1,-1}
k=1
k=0.95
k=1000
emf_k2
integrator2
k=0.95
k=1



- $\mathbf{0} = \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}, t)$
- $\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{y}, \mathbf{x}) \\ \mathbf{0} = \mathbf{g}(\mathbf{y}, \mathbf{x}) \end{cases}$
- Compositional design
- Tools: Dymola (Dassault Systèmes), Modelica

`if` **Guard** <u>`do`</u> **Differential Equation**

- **Guard**: predicate in the state variables and their **time derivatives**.
- **Differential Equation**: equation, **implicit** or explicit, in the state variables and their time derivatives.

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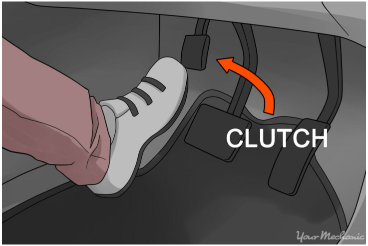
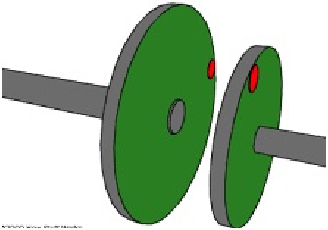
When a guard holds, its equation is enforced.

Ideal Clutch





$$
\begin{array}{rlll}
\texttt{if t} & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 & (e_1) \\
\texttt{if t} & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 & (e_2) \\
\texttt{if } \gamma & \texttt{do} & \omega_1 - \omega_2 = 0 & (e_3) \\
\texttt{if } \gamma & \texttt{do} & \tau_1 + \tau_2 = 0 & (e_4) \\
\texttt{if } \neg\gamma & \texttt{do} & \tau_1 = 0 & (e_5) \\
\texttt{if } \neg\gamma & \texttt{do} & \tau_2 = 0 & (e_6)
\end{array}
$$

- **State Variables**: the angular velocities ω_1 and ω_2
- γ is an input signal modelling the pedal's position

$$
\begin{array}{lllr}
\texttt{if } \mathsf{t} & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 & (e_1) \\
\texttt{if } \mathsf{t} & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 & (e_2) \\
\texttt{if } \gamma & \texttt{do} & \omega_1 - \omega_2 = 0 & (e_3) \\
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`if t`	`do`	$J_1\dot{\omega}_1 = \tau_1$	(e_1)
`if t`	`do`	$J_2\dot{\omega}_2 = \tau_2$	(e_2)
`if` γ	`do`	$\omega_1 - \omega_2 = 0$	(e_3)
`if` γ	`do`	$\tau_1 + \tau_2 = 0$	(e_4)
`if` $\neg\gamma$	`do`	$\tau_1 = 0$	(e_5)
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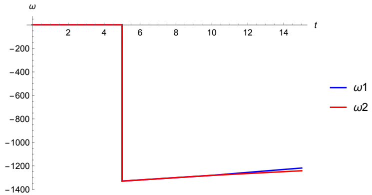
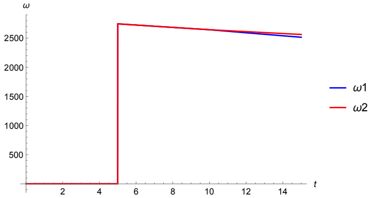
- Dymola **crashes** with a division by zero
- Mathematica treats resets as initializations (nondeterministic behavior)





The solution may be discontinuous when γ : f $\rightarrow$ t because of the additional constraint $\omega_1 - \omega_2 = 0$

Problem 1 How to handle overdetermined systems ?

- The angular velocities ω_1 and ω_2 are **known**
- γ switches to t (the driver engages the clutch)

$$\omega_1 - \omega_2 = 0 \text{ is } \textbf{enforced}$$

- The system **becomes** overdetermined
- The solution is not smooth and even discontinuous

Problem 2 What is the meaning of the derivatives ?

Some equations must hold for $\gamma = \mathsf{t}$ and $\gamma = \mathsf{f}$.

$$\begin{array}{lll} \texttt{if t do} & J_1\dot{\omega}_1 = \tau_1 & (e_1) \\ \texttt{if t do} & J_2\dot{\omega}_2 = \tau_2 & (e_2) \end{array}$$

- What is the **meaning** of derivatives when $\gamma : \mathsf{f} \to \mathsf{t}$?
- How to compute the **reset values** ?

Causality Principle

The additional constraints are

- **caused** by (consequence of) the **current** status, and
- **enforced** at the **immediate next** instant

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t : **present**

$\omega_1(t) - \omega_2(t) \neq 0$

$\omega_1(t+\delta) - \omega_2(t+\delta) = 0$

$t + \delta \, , \; 0 < \delta << 1$: **immediate future**

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$t + \delta\ ,\ 0 < \delta << 1$: **immediate future**

$\delta \in {}^\star\mathbb{R}$ is a **positive infinitesimal**

- $\delta = \langle \delta_1, \delta_2, \dots \rangle$
- $\delta_i \in \mathbb{R}$
- Not necessarily convergent
- $\langle 1, \frac{1}{2}, \frac{1}{3}, \dots \rangle$ is a (positive) infinitesimal
- $r = \langle r, r, r, \dots \rangle$, $r \in \mathbb{R}$
- Functions over the reals can be *internalized*
- $x(\langle t_1, t_2, \dots \rangle) = \langle x(t_1), x(t_2), \dots \rangle$

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Let $\delta \in {}^\star\mathbb{R}$ be a non zero infinitesimal.

$$\frac{x(t+\delta)-x(t)}{\delta}$$

Proposition

A real function x is differentiable at t if and only if there exists a real number b such that

$$\frac{x(t+\epsilon)-x(t)}{\epsilon} \sim b$$

for any non zero infinitesimal ϵ.

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$$\dot{x} \text{ is replaced by } \frac{x(t+\delta) - x(t)}{\delta} = \frac{x^{\bullet} - x}{\delta}$$

- **Shift forward** (when needed)
- **Formal substitution** of time derivatives into difference quotients.

$$\begin{array}{rll}
\texttt{if t} & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 \quad (e_1) \\
\texttt{if t} & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 \quad (e_2) \\
\texttt{if } \gamma & \texttt{do} & \omega_1 - \omega_2 = 0 \quad (e_3) \\
\texttt{if } \gamma & \texttt{do} & \tau_1 + \tau_2 = 0 \quad (e_4) \\
\texttt{if } \neg\gamma & \texttt{do} & \tau_1 = 0 \quad (e_5) \\
\texttt{if } \neg\gamma & \texttt{do} & \tau_2 = 0 \quad (e_6)
\end{array}$$

$$\begin{array}{rll}
\texttt{if t} & \texttt{do} & J_1\frac{\omega_1^\bullet - \omega_1}{\delta} = \tau_1 \quad (e_1^\delta) \\
\texttt{if t} & \texttt{do} & J_2\frac{\omega_2^\bullet - \omega_2}{\delta} = \tau_2 \quad (e_2^\delta) \\
\texttt{if } \gamma & \texttt{do} & \omega_1^\bullet - \omega_2^\bullet = 0 \quad (e_3^\bullet) \\
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$$
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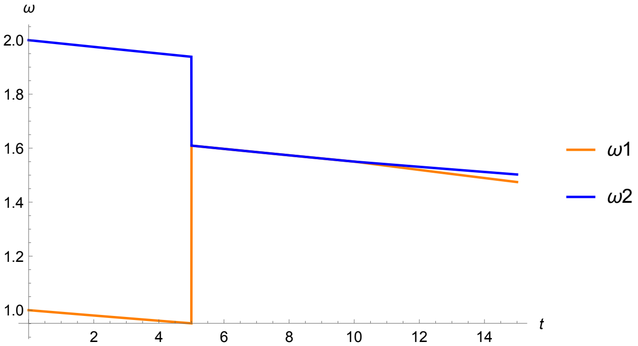
\end{array}
$$

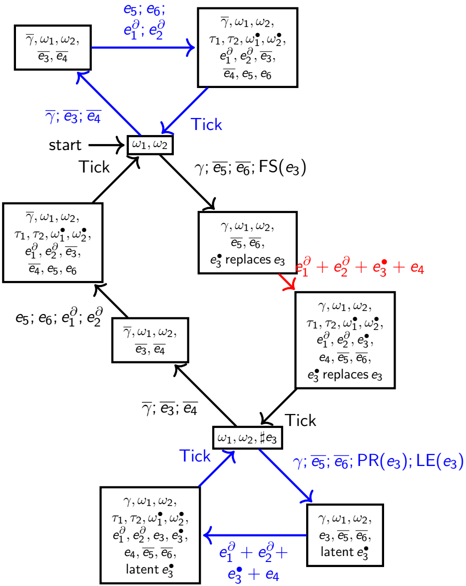
$$\omega_1^\bullet = \omega_2^\bullet = \frac{J_1\omega_1 + J_2\omega_2}{J_1 + J_2}$$

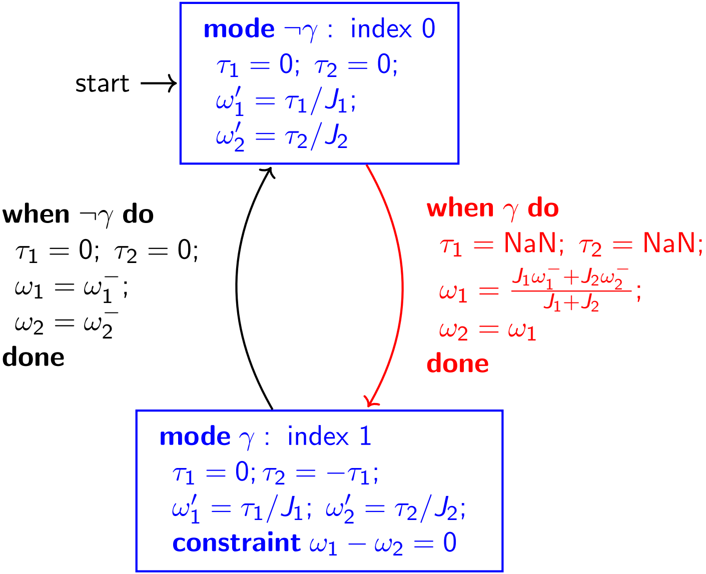
Standardization

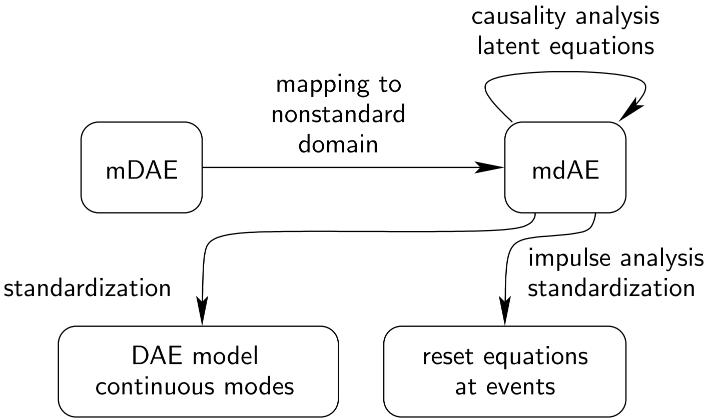
- Automated procedure for a class of systems
- Generalization remains a challenge

Expected Simulation




$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
start
ω_1, ω_2
Tick
$\gamma; \overline{e_5}; \overline{e_6}; \mathsf{FS}(e_3)$
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\gamma, \omega_1, \omega_2, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$e_1^\partial + e_2^\partial + e_3^\bullet + e_4$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$\gamma, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, e_3^\bullet, e_4, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
$\omega_1, \omega_2, \sharp e_3$
Tick
$\gamma; \overline{e_5}; \overline{e_6}; \mathsf{PR}(e_3); \mathsf{LE}(e_3)$
$\gamma, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, e_3, e_3^\bullet, e_4, \overline{e_5}, \overline{e_6}$, latent $e_3^\bullet$
$e_1^\partial + e_2^\partial + e_3^\bullet + e_4$
$\gamma, \omega_1, \omega_2, e_3, \overline{e_5}, \overline{e_6}$, latent $e_3^\bullet$


start
mode $\neg\gamma$: index 0
$\tau_1 = 0;\ \tau_2 = 0;$
$\omega_1' = \tau_1/J_1;$
$\omega_2' = \tau_2/J_2$
when $\neg\gamma$ **do**
$\tau_1 = 0;\ \tau_2 = 0;$
$\omega_1 = \omega_1^-;$
$\omega_2 = \omega_2^-$
done
when γ **do**
$\tau_1 = \mathrm{NaN};\ \tau_2 = \mathrm{NaN};$
$\omega_1 = \frac{J_1\omega_1^- + J_2\omega_2^-}{J_1 + J_2};$
$\omega_2 = \omega_1$
done
mode γ : index 1
$\tau_1 = 0; \tau_2 = -\tau_1;$
$\omega_1' = \tau_1/J_1;\ \omega_2' = \tau_2/J_2;$
constraint $\omega_1 - \omega_2 = 0$


causality analysis
latent equations
mapping to
nonstandard
domain
mDAE
mdAE
standardization
impulse analysis
standardization
DAE model
continuous modes
reset equations
at events

Thanks for your attention !