Adapting Neural Networks to Fixed-Point and Integer Arithmetic

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Summary

Plan

Introduction

Difficulty to run tasks in real time

Embedded systems have often low memory

Difficulty maintaining efficiency with limited resources

New Problem:

Artificial Intelligence and especially Neural Networks are increasingly used in Embedded Systems !!!

Contribution

Input: A trained NN, working at some precision (Floating-Point Format)

Output: Code generation (the new NN) to "simulate" the original NN using integers and fixed-point arithmetic

Correctness: The new NN with smaller data types behaves almost like the original NN

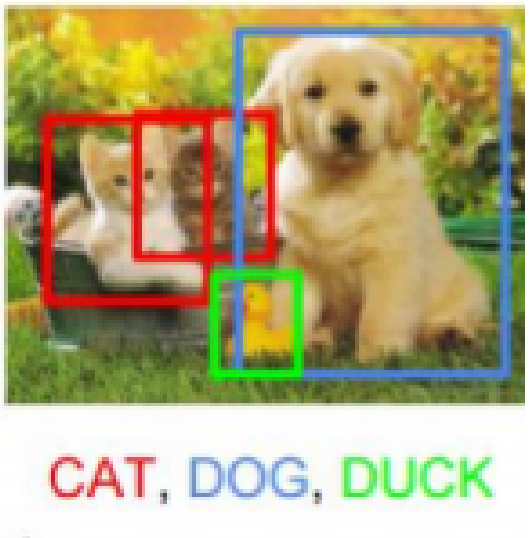
- Function Approximation: results stay in a desired hull
- Classification : only x% decisions differ from original NN

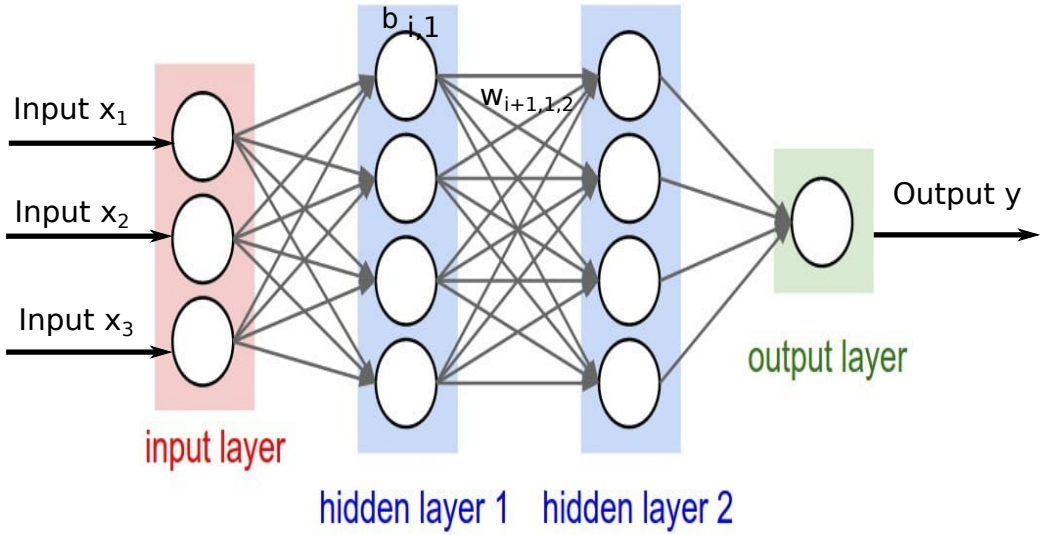
"Simulate": **Same behavior as the original network with x% error**

Plan

Neural Networks (NN)

- The conception is inspired by the biological functioning of neurons
- NN create fast classifications and generalizations
- NN are used on a variety of tasks (object recognition, speech and handwriting recognition, machine translation, medical diagnosis...)





Plan

Fixed-Point Format (FPF)

Float numbers are represented by **integers** in a format <m,l> or <I,F>

Definition

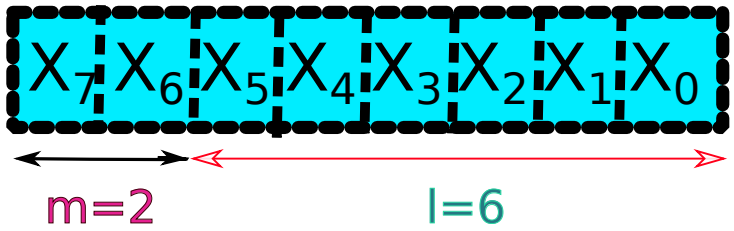
For $X = (x_{n-1}x_{n-2}...x_1x_0)_\beta$ and $x = X.\beta^{-l}$, we have: $p = m + l + 1$.

x : Fixed-Point number with implicit scale factor β^{-l},

X: Integer representation, p : Word **length**, m : The **most** significant bit,

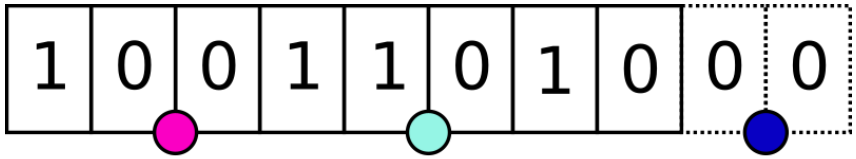
l : The **least** significant bit

Representation of X in a format <m,l>=<2,6>



⚠ **No exponent, the programmer must take care of the formats !**

Different interpretations of the same integer representation



A Fixed-Point number in different format

Scaling Factor Symbol	Format	Value of X	Value of x
purple	< 2, 6 >	$(10011010)_2 = (154)_{10}$	$(10.011010)_2 = (2.40625)_{10}$
cyan	< 5, 3 >	$(10011010)_2 = (154)_{10}$	$(10011.010)_2 = (19.25)_{10}$
blue	< 9, -1>	$(10011010)_2 = (154)_{10}$	$(100110100)_2 = (308)_{10}$

Representation of 154 in different formats

Plan

Plan

Addition

Let : x_1 and x_2 FP numbers with FPF <i, f> :

$$x_1 + x_2 = x_3$$

$$< i, f > \; + \; < i, f > \; = \; < i, \; f > \tag{1}$$

$$\begin{cases} 0 < i < p \\ 0 < i + f < p \\ f = p - i \end{cases} \tag{2}$$

Example :

$$\begin{array}{rl} & 000000.010 \quad <6, 3> \\ + & \\ & 001000.000 \quad <6, 3> \\ \hline & 001000.010 \quad <6, 3> \end{array}$$

Multiplication

Let : x_1 and x_2 FP numbers with FPF <i, f> :

$$x_1 \times x_2 = x_3$$

$$< i, f > \times < i, f > = < i - f, 2f > \tag{3}$$

$$\begin{cases} 0 < i < p \\ 0 < i + f < p \\ f = p - i \end{cases} \tag{4}$$

Shift Right

$$x_1 >> k = x_2$$

$$< i, f > \; >> \; k \; = \; < i + k, \; f - k > \tag{5}$$

$$\begin{cases} 0 < i < p \\ 0 < i + f < p \\ f = p - i \end{cases} \tag{6}$$

Shift Left

$$x_1 << k = x_2$$

$$< i, f > \; << \; k \; = \; < i - k, \; f + k > \tag{7}$$

$$\begin{cases} 0 < i < p \\ 0 < i + f < p \\ f = p - i \end{cases} \tag{8}$$

AND,OR, XOR

$$x_1 \ \ AND \ \ x_2 = x_3$$

$$< i, f > \ AND \ < i, f > \ = \ < i, \ f > \tag{9}$$

$$\begin{cases} 0 < i < p \\ 0 < i + f < p \\ f = p - i \end{cases} \tag{10}$$

Same format for OR and XOR !

Plan

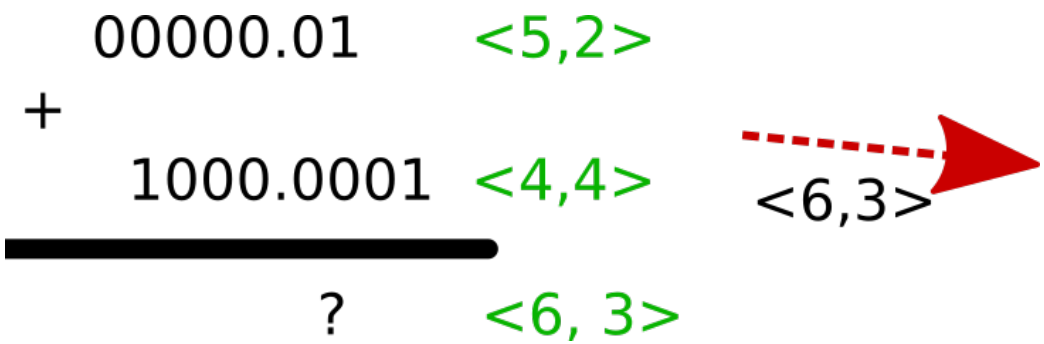
Addition

$$x_1 + x_2 = x_3$$

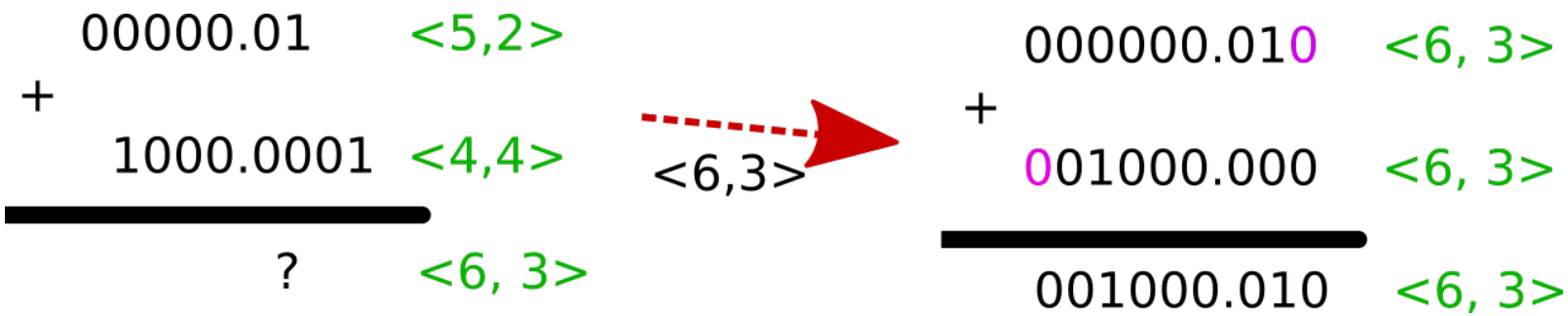
$$< i_1, f_1 > \ + \ < i_2, f_2 > \ = \ < i_3, \ f_3 > \tag{11}$$

$$\begin{cases} 0 < i_3 < p \\ 0 < i_3 + f_3 < p \\ i_3 = max(i_1, i_2) \\ f_3 = p - i_3 \end{cases} \tag{12}$$

Example 1: fixed format

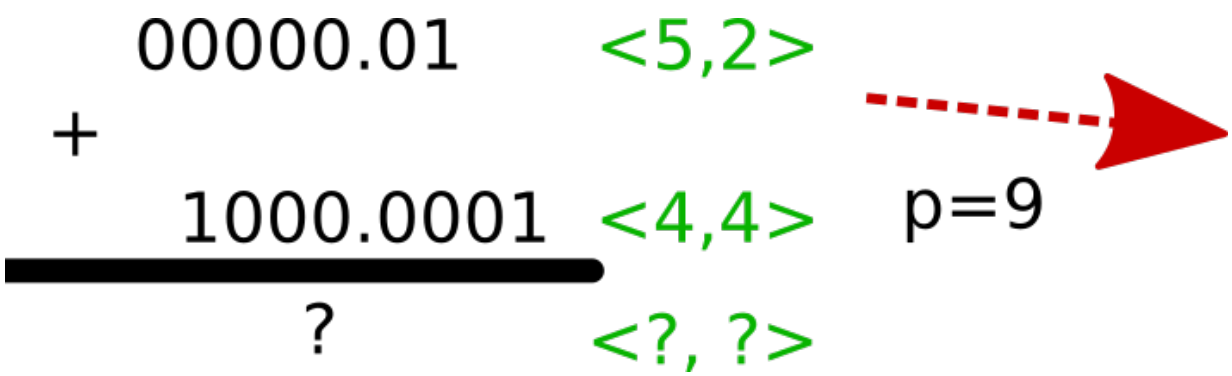


Example 1

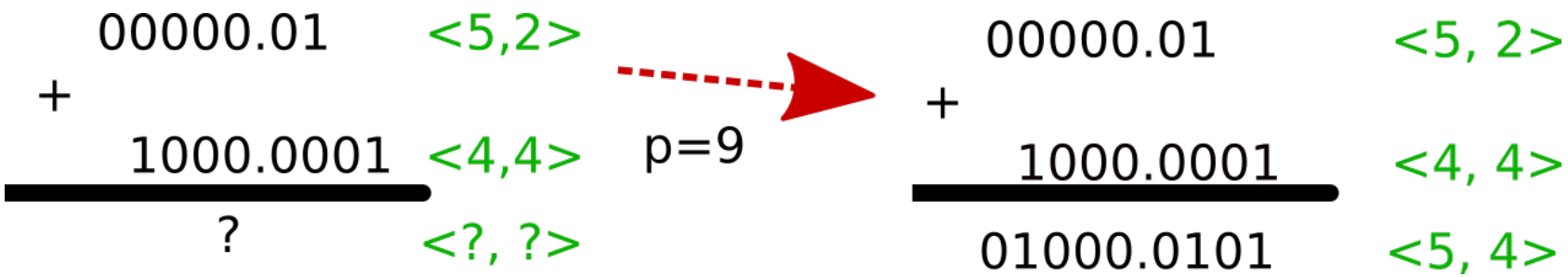

00000.01 <5,2>
+
1000.0001 <4,4>
?
<6, 3>
<6,3>
000000.010 <6, 3>
+
001000.000 <6, 3>
001000.010 <6, 3>

Example 2

Example 2: fixed precision



Example 2


00000.01 <5,2>
+
1000.0001 <4,4>
?
<?, ?>
p=9
00000.01
<5, 2>
+
1000.0001
<4, 4>
01000.0101
<5, 4>

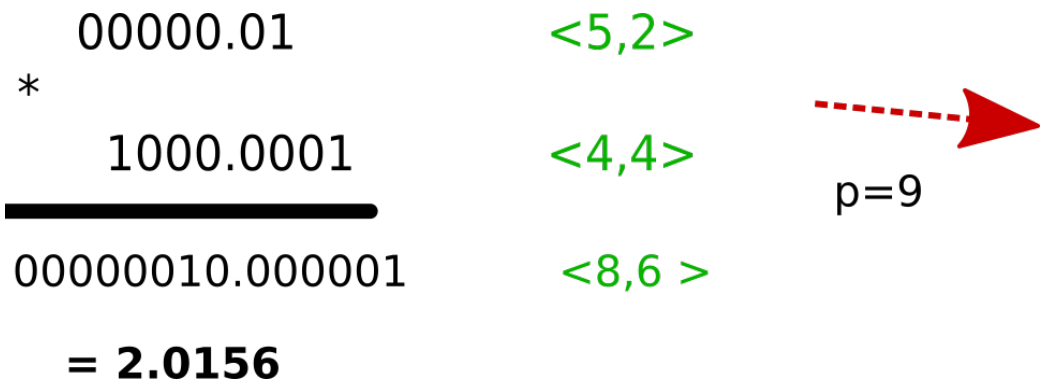
Multiplication

$$x_1 \times x_2 = x_3$$

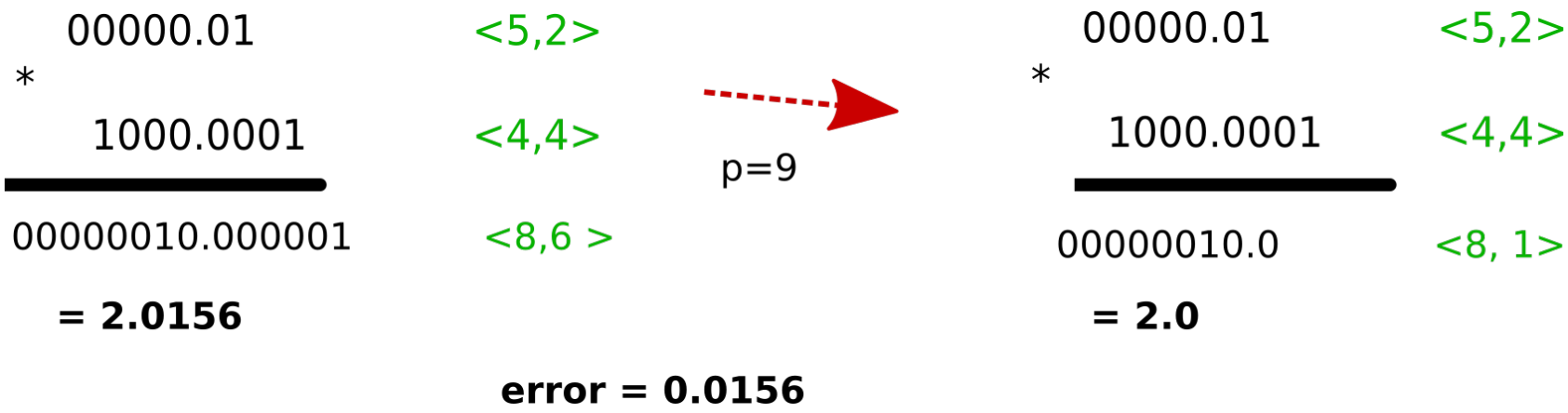
$$< i_1, f_1 > \times < i_2, f_2 > = < i_3, f_3 > \tag{13}$$

$$\begin{cases} 0 < i_3 < p \\ 0 < i_3 + f_3 < p \\ i_3 = i_1 + i_2 - 1 \\ f_3 = p - i_3 \end{cases} \tag{14}$$

Example: fixed precision



Example


00000.01
<5,2>
*
1000.0001
<4,4>
00000010.000001
<8,6 >
= 2.0156
p=9
00000.01
<5,2>
*
1000.0001
<4,4>
00000010.0
<8, 1>
= 2.0
error = 0.0156

AND, OR, XOR

$$x_1 \ \ AND \ \ x_2 = x_3$$

$$< i_1, f_1 > \ AND \ < i_2, f_2 > \ = \ < i_3, \ f_3 > \tag{15}$$

$$\begin{cases} 0 < i_3 < p \\ 0 < i_3 + f_3 < p \\ i_3 = max(i_1, i_2) \\ f_3 = p - i_3 \end{cases} \tag{16}$$

Same format for OR and XOR !

Plan

Format of weighted sum

Let :

$$f(\hat{x}) = W\hat{x} + b$$

$$f(\hat{x}) = \sum_{j=0}^{n} w_{ij}\,\hat{x}_j + b_i$$

$$f(\hat{x}) = \underbrace{\underbrace{\underbrace{w_{i1}\,\hat{x}_1}_{<i_2,f_2> \times <i_1,f_1>} + \underbrace{w_{i2}\,\hat{x}_2}_{<i_2,f_2> \times <i_1,f_1>} + \ldots + \underbrace{w_{in}\,\hat{x}_n}_{<i_2,f_2> \times <i_1,f_1>}}_{<i_1+i_2-1,f_1+f_2> \; + \; <i_1+i_2-1,f_1+f_2> \; + \; <i_1+i_2-1,f_1+f_2>} + \underbrace{b_i}_{<i_3,f_3>}}_{<i_4,f_4> \; = \; <i_1+i_2-1,f_1+f_2>}$$

$$<i_5,f_5> \; = \; <max(i_4,i_3),p-max(i_4,i_3)> \; = \; <max((i_1+i_2-1),i_3),p-max((i_1+i_2-1)>$$

$$\boxed{<\mathbf{i_5},\mathbf{f_5}> \; = \; <\mathbf{max}((\mathbf{i_1}+\mathbf{i_2}-\mathbf{1}),\mathbf{i_3}),\mathbf{p}-\mathbf{max}((\mathbf{i_1}+\mathbf{i_2}-\mathbf{1})>} \tag{17}$$

W : matrix of weights, $\hat{\mathbf{x}}$: inputs and **b** : bias

$<\mathbf{i_1},\mathbf{f_1}>$: representation format of the input vector $\hat{x}$

$<\mathbf{i_2},\mathbf{f_2}>$: representation format of W

$<\mathbf{i_3},\mathbf{f_3}>$: representation format of bias b

$<\mathbf{i_5},\mathbf{f_5}>$: representation format of the result $f(\hat{x})$

Plan

Conclusion & Perspectives

- Reducing the size and execution time of large NN with this approach
- Implementing a code transformation tool
- Experiments on large NN
- Handling classifiers and interpolators.

Related Work

Dutta S. Jha S., Sankaranarayanan S. Tiwari A. (2018). "Output Range Analysis for Deep Feedforward Neural Networks". In: *NFM*.

Gehr T. Mirman M., Drachsler-Cohen D. Tsankov P. Chaudhurri S. Vechev M.T. (2018). "AI^2: Safety and Robustness Certification of Neural Networks with Abstract Interpretation". In: *SP*.

Martel, Matthieu (2017). "Floating-Point Format Inference in Mixed-Precision". In: *NFM*.

Singh G. Gehr T., Puschel M. Vechev M.T. (2019). "An Abstract Domain for Certifying Neural Networks". In: *POPL*.

- Gehr T. 2018 Singh G. 2019

- Gehr T. 2018 Singh G. 2019
- Martel 2017
- Dutta S. 2018