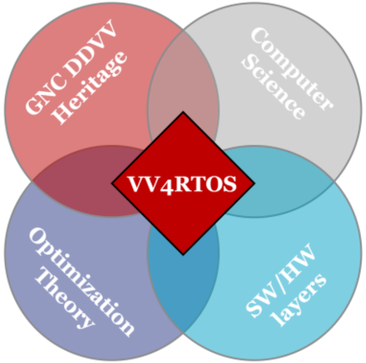
V&V of Optimization-based Control Systems

Developments and Objectives of the ESA VV4RTOS Project



GMV

TAS-F

ENAC (LII)

IMTEK (SYSCOP)

Pedro Lourenço
Head of Guidance & Control Section
GNC Division
Flight Segment and Robotics BU

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The project

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Verification & Validation

Optimization Software

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V&V of G&C

The team

A global high technology group

Multinational technology group

6th European Space Industrial Group

2,500+ employees

Roots tied to Space

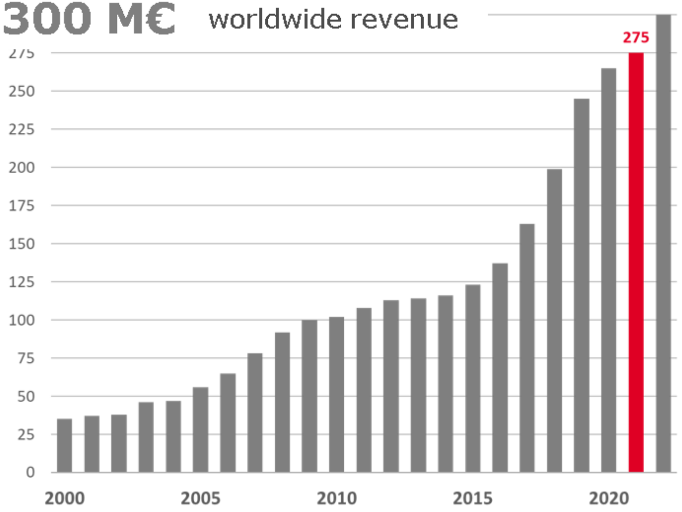
CMMI level 5

Private capital

Founded in **1984**

Companies in 12 countries

300 M€ worldwide revenue



Space, Aeronautics, Defense & Security, Intelligent Transportation, Banking & Finances, ICT Industries

Space	Defense	IT	Transport
57%	10%	17%	16%

Flight Segment and Robotics

Specialized Team in Portugal:

- Guidance Navigation and Control Algorithms
- Integrated GNC+Avionics System Architecture & Design
- ATB/AIV Qualification / V&V
- On Board Software OBSW
- Integrated Modular Avionics / Time Space Partition
- Robotics

VV4RTOS team

Organization

Funding and coordination
Technical Officer: Dr. Valentin Preda

Project Consortium
Project Manager: Dr. Pedro Lourenço
GNC Engineers: Hugo Costa
Pedro Cachim
SW Engineers: Carolina Serra
Emanuel Ferreira

V&V experts: Prof. Pierre-Loïc Garoche
Dr. Arash Sadeghzadeh

Optimization Experts: Dr. Gianluca Frison
Jonathan Frey

System experts: Dr. Anthea Comellini

Responsibilities

GMV:
- Project coordination
- V&V gaps
- V&V, FES, G&C implementation

Thales Alenia Space:
- Support V&V gaps
- V&V execution
  - SIL/PIL

République Française / ENAC:
- Formal V&V expertise

IMTEK:
- Optimization theory
- Optimization software

The context

Launchers

Satellites

Interplanetary missions

Space Robotics

Rendezvous & Docking

OSAM

Active Debris Removal

…

A conservative Industry

Controllers flying over the years

1960 — 1990-2000 — 1990-2000 — 2004-2011 — 1998-2014 →

1960

US Saturn V

Rigid: PID
Flexible: Filters

Manual gain-sched.

1990-2000

Japan M-V

Rigid+Flexible:
H∞ Mixed Sensitivity

Manual gain-sched.

1990-2000

EU Ariane 5 Plus

Rigid: H∞ Mixed Sen
Flexible: LQG

Manual gain-sched.

2004-2011

US Ares-I

Rigid: PID
Flexible: Filters

Filters tuned through
Constrained Optimization

1998-2014

EU VEGA

Rigid: PID
Flexible: Filters

Manual gain-sched.

Current and future trends

New tools being considered

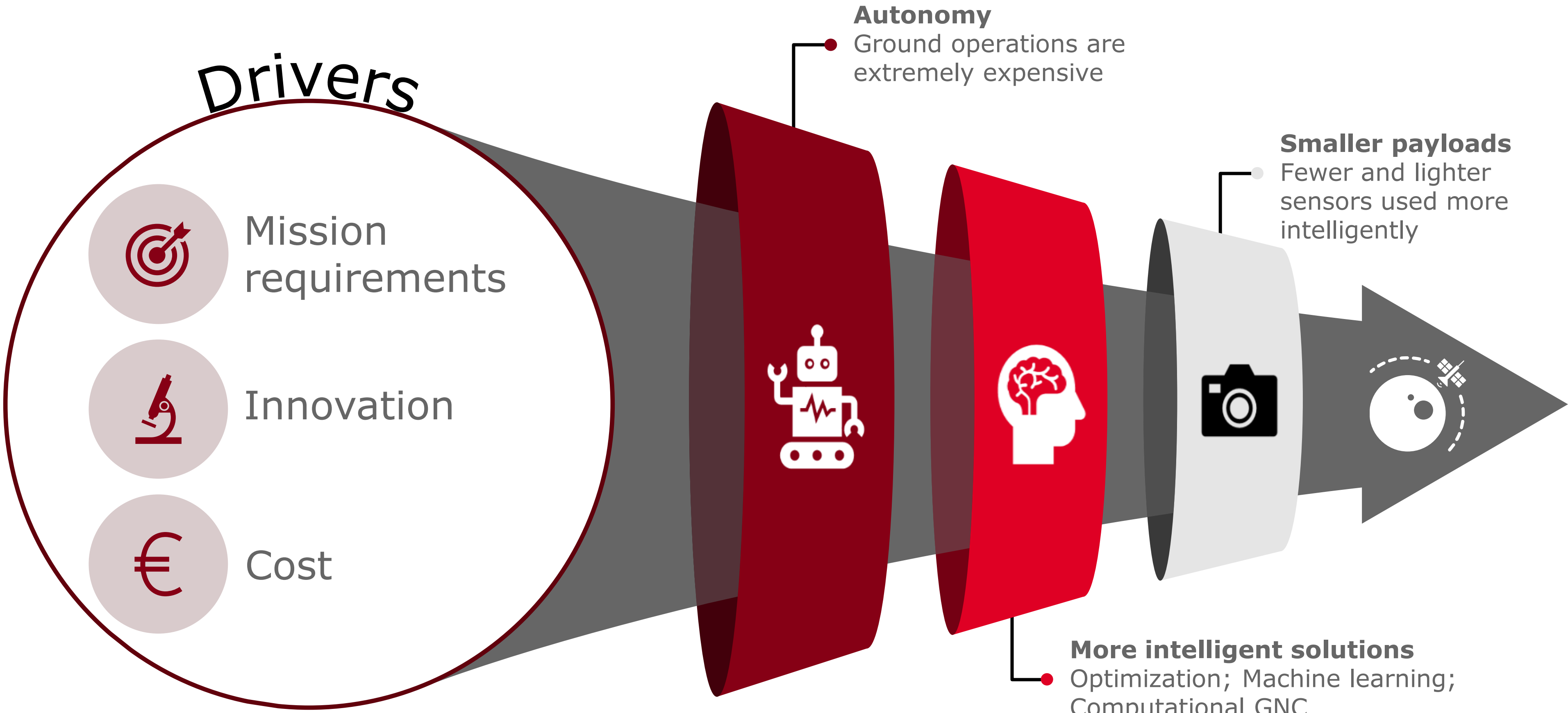
New concepts and challenges in space

New space

- Democratization of space, commercial access to launches, with micro- and nano-satellites

New challenges

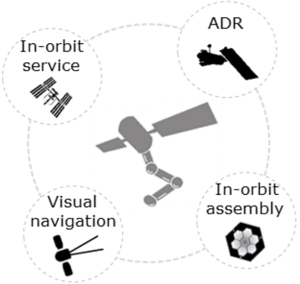
- Cost reduction
- Debris removal



Drivers
Mission requirements
Innovation
Cost
Autonomy
Ground operations are extremely expensive
Smaller payloads
Fewer and lighter sensors used more intelligently
More intelligent solutions
Optimization; Machine learning; Computational GNC

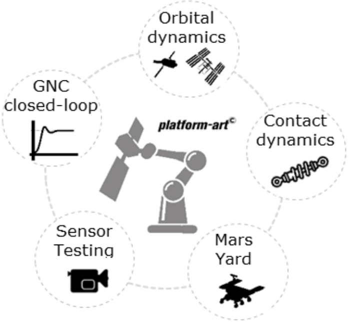
Verification & Validation

Overview



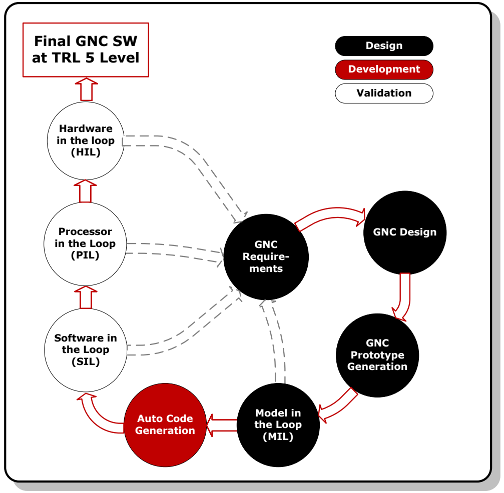
Orbital Robotics

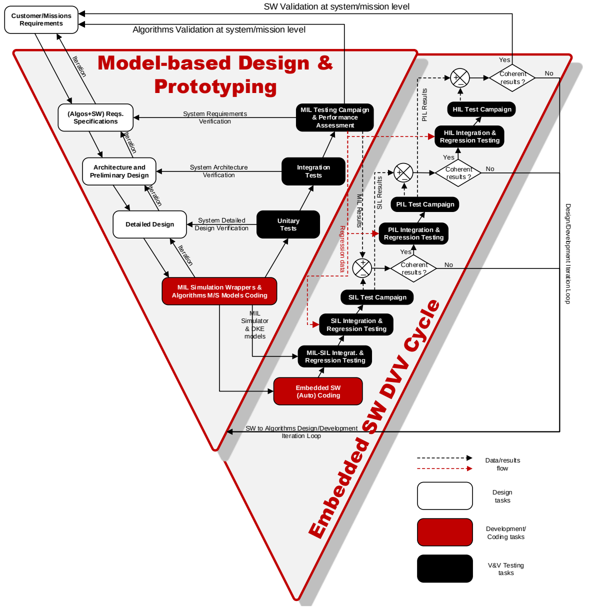
- Advanced control for capture & detumbling of debris
- Visual navigation & inspection in-orbit
- Robotic assembly of large and flexible structures
- Robotic in-orbit servicing & refueling



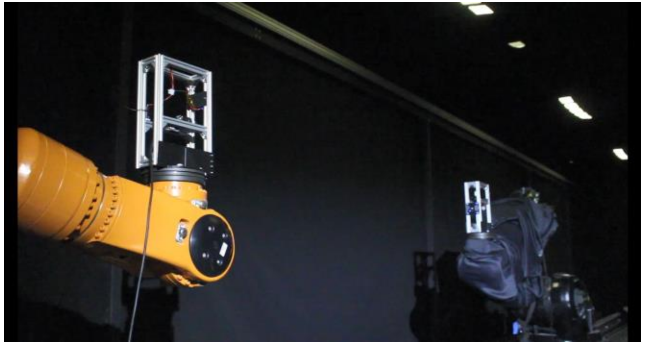
Simulation & Test Facilities

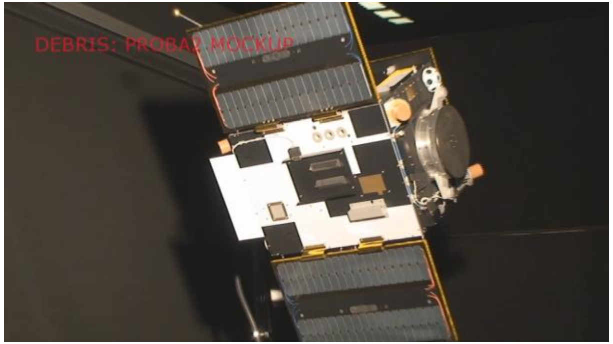
- Orbital dynamics simulation for RdV and FF
- Navigation sensors test
- GNC closed loop experiments
- Contact dynamics (ADR, in-orbit assembly)
- Planetary robotics test campaigns

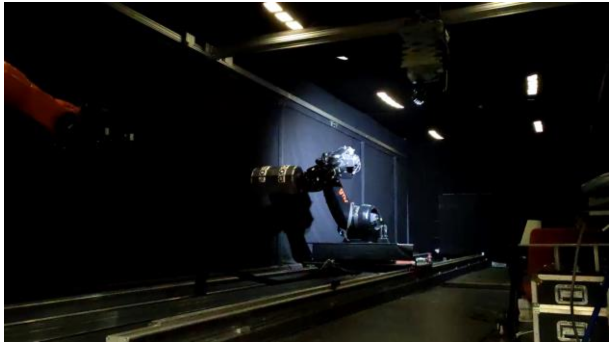


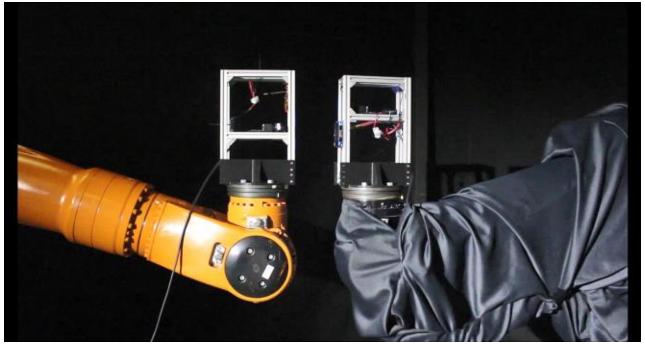


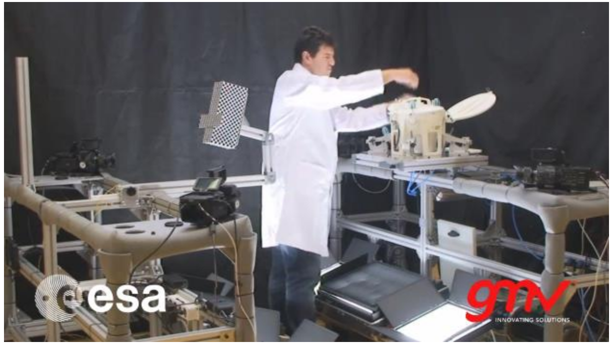
Hardware-in-the-loop









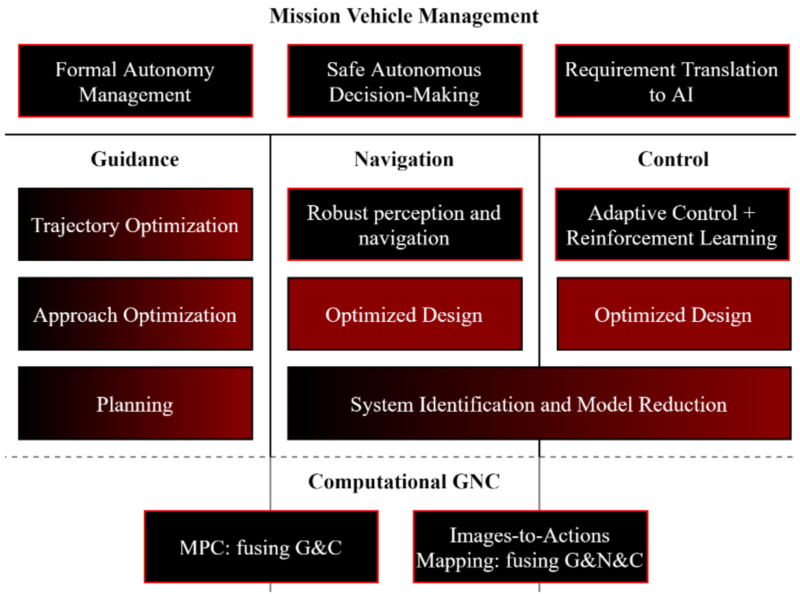


The future of the industry

Up-and-coming tools and technologies

- On-board Optimization
- Machine Learning
- Robust G&C
- Image Processing, SDR
- Autonomy

Computational G&C



Challenges

- Computational cost of the algorithms
- Robustness and stability guarantees
- Efficient and representative V&V

The project

VV4RTOS

Verification and Validation of Real-Time Optimised Safety-Critical GNC SW Systems

VV4RTOS

Objectives

"The activity aims to define optimisation architectures, GNC and real-time optimisation algorithms, and verification & validation (V&V) processes that guarantee safe code execution under resource and timing constraints."

Verification & Validation

- Augment traditional GNC DDVV to explicitly address iterative embedded optimization algorithms
- Guarantee safe, reliable, repeatable, and accurate execution of the OBSW

Optimization-based G&C

- Consolidate process for fast prototyping and qualification of G&C SW
- Theoretical foundations for optimization problem posing, discretization, convexification, and transcription into online-solvable programs.
- High-to-low level translation of mission requirements and interface with certified embedded solvers

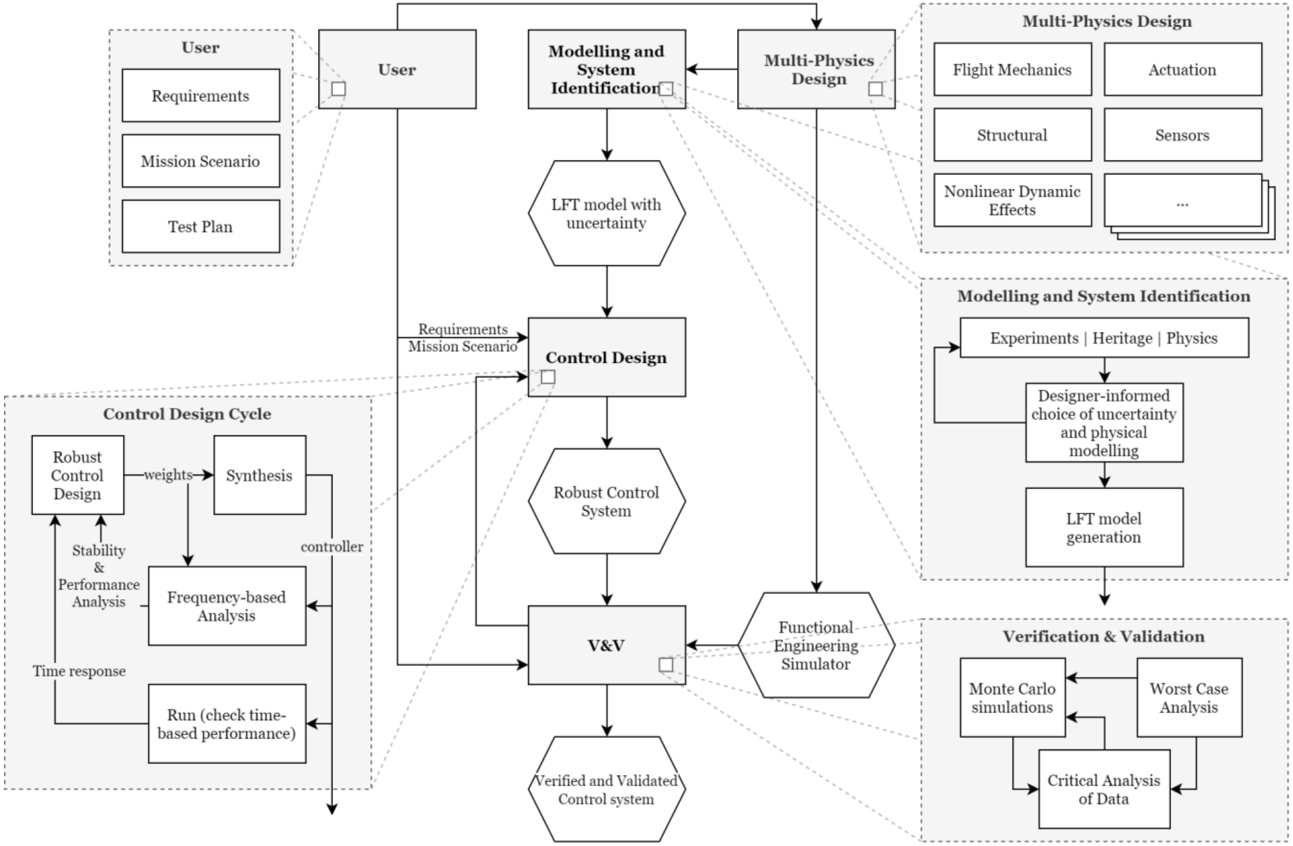
Technical approach

VV4RTOS

Verification and Validation of Real-Time Optimised Safety-Critical GNC SW Systems

Technical approach

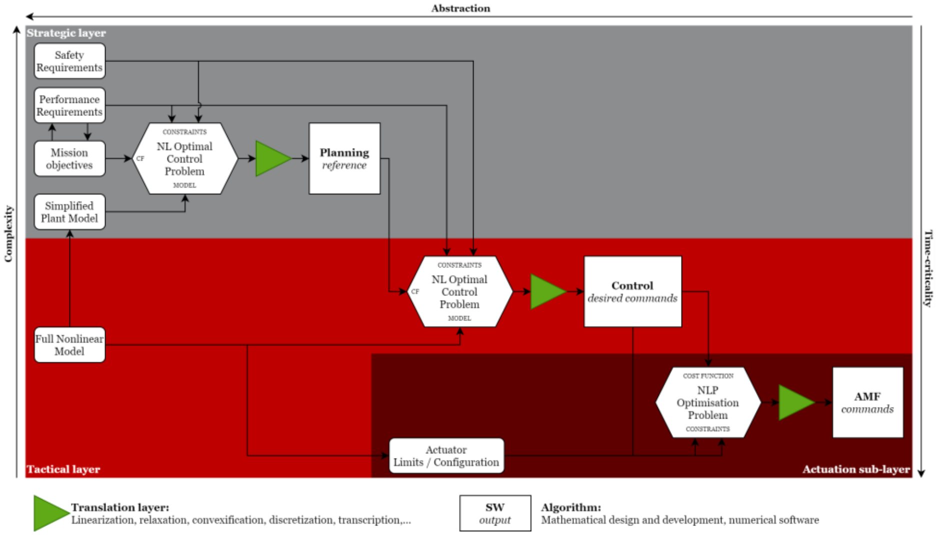
The “old” ways



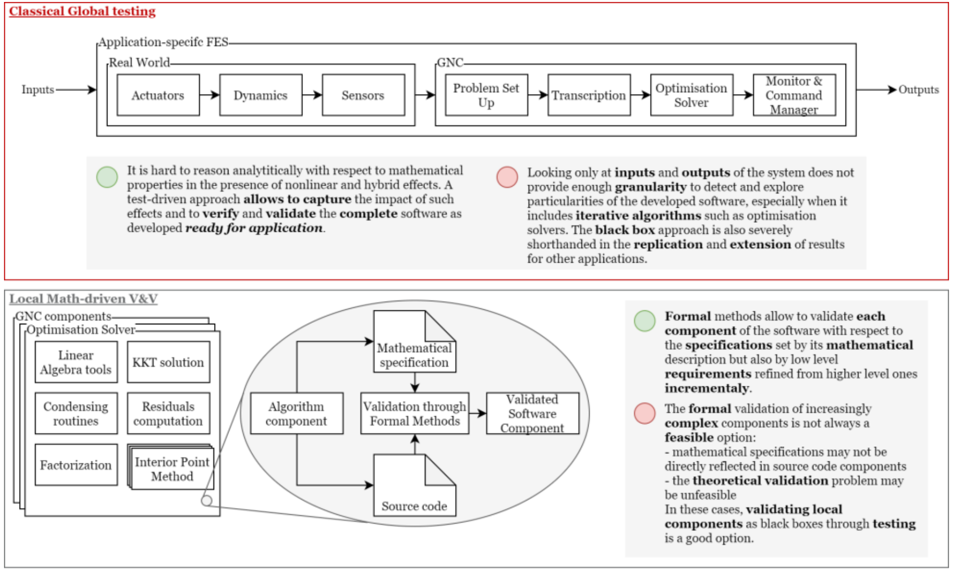
Technical approach

Objectives

Optimization-based G&C



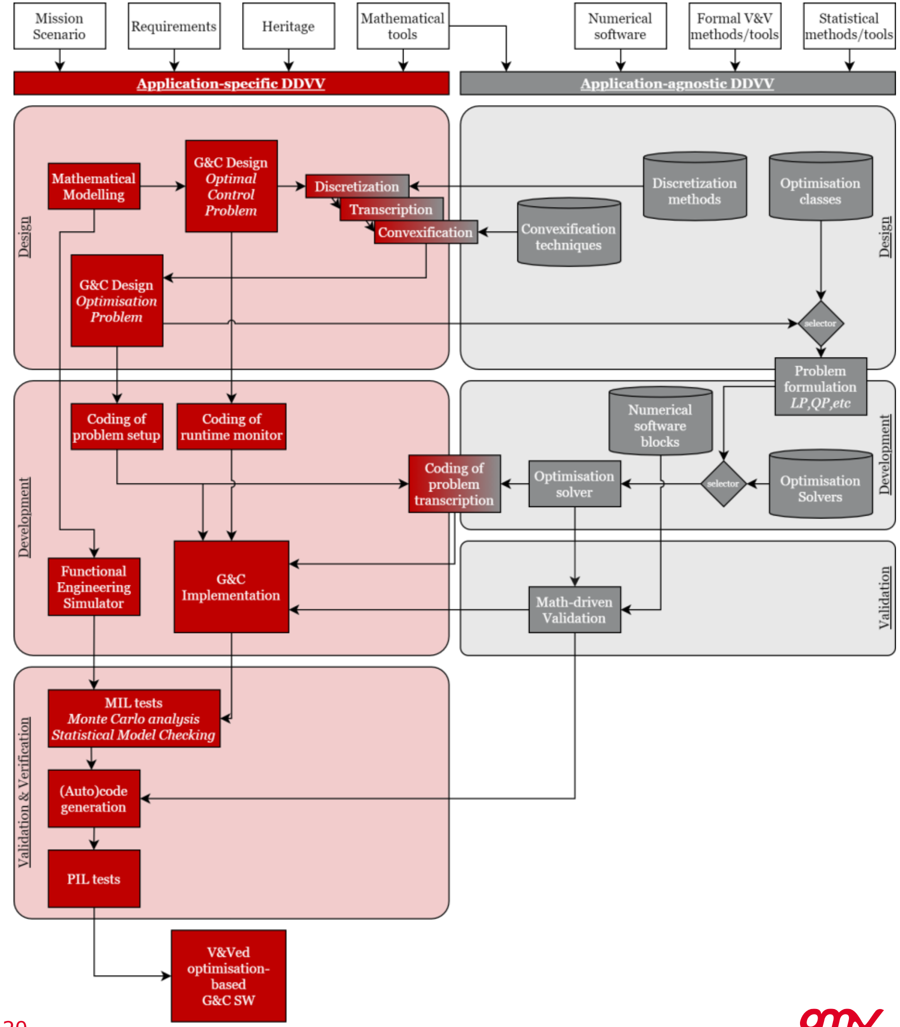
Enhanced V&V



Technical approach

Objectives

Mixed Formal and Testing Approach to the DDVV of optimisation-based G&C software



Technical approach

Verification & Validation

Usecase

- Guidance & Control problem
- First-order methods vs Interior point methods

Requirements

- Identify meaningful requirements
- Mathematical properties of the algorithms

Means of compliance

- Classical means: testing
- Formal specification and validation

Optimization

Classification

Optimization problems

Problem type:
- QP, QCQP, SOCP
- Convexity
- (mixed integer)

Problem structure:
- Dense, Sparse
- Optimal control structure

Problem properties:
- Numerical properties (e.g., conditioning)
- Sampling time
- Availability of initial guess

Optimization solvers

Solver type:
- First order
- Active set
- Interior Point

Solver properties:
- Speed, accuracy, robustness
- Average vs worst case solution time
- Single/double precision floating/fix point
- Memory (amount, static/dynamic)
- Warm start capabilities
- Code complexity
- Ease of V&V (e.g., theoretical bounds)

Analysis of Numerical Optimization Software

First Order Methods

- ✓ Simple and concise code
- • Many but cheap iterations
- ❑ Slow convergence to high accuracy
- ❑ Sensitive to scaling
- ✓ Need low floating/fixed point accuracy
- ✓ Existence of practically relevant convergence bounds
- ✓ "Easy" to formally V&V

Y. Yu, P. Elango, U. Topcu, and B. Açıkmeşe, "Proportional-Integral Projected Gradient Method for Conic Optimization," Automatica, vol. 142, p. 110359, Aug. 2022, doi: 10.1016/j.automatica.2022.110359.

Interior Point Methods

- ❑ Complex code (e.g., factorizations)
- • Few but expensive iterations
- ✓ Fast convergence to high accuracy
- ✓ Unsensitive to scaling (Newton)
- ❑ Need high FP accuracy (ill-condition)
- ❑ No practically relevant convergence bounds for state-of-the-art methods
- ❑ Difficult to formally V&V

https://github.com/giaf/hpipm

G. Frison, J. Frey, F. Messerer, A. Zanelli, and M. Diehl, "Introducing the quadratically-constrained quadratic programming framework in HPIPM," in 2022 European Control Conference (ECC), London, United Kingdom, Jul. 2022, pp. 447–453. doi: 10.23919/ECC55457.2022.9838499.

HPIPM Analysis

HPIPM

Language	files	blank	comment	code
C	325	20784	25466	130221
C/C++ Header	126	2292	6658	5983
MATLAB	69	1409	1874	4796
Python	22	584	1095	1554
make	20	184	788	756
Mathematica	1	25	0	533
CMake	3	44	157	267
TeX	2	56	81	244
Julia	8	37	16	183
Markdown	8	34	0	179
YAML	1	9	0	166
Bourne Shell	5	32	45	108
Bourne Again Shell	1	6	27	2
SUM:	591	25496	36207	144992

BLASFEO

Language	files	blank	comment	code
Assembly	148	135341	156748	331162
C	275	44874	24305	202707
make	35	1468	1435	4539
C/C++ Header	49	1117	3176	4027
CMake	33	403	381	1840
JSON	31	30	0	1806
MATLAB	6	468	62	1132
Python	2	149	42	343
Markdown	4	88	0	289
Bourne Shell	7	7	30	270
TeX	2	88	11	168
YAML	1	12	0	145
SUM:	593	184045	186190	548428

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Analysis of Numerical Optimization Software

General conic optimization problem

Conic optimization problem is the minimization of a differentiable convex objective function f subject to conic constraints:

$$\begin{aligned} \min_{z,y} \quad & f(z) \\ \text{s.t.} \quad & Hz - y = g, \quad y \in \mathbb{K}, \quad z \in \mathbb{D} \end{aligned} \tag{1}$$

where $z \in \mathbb{R}^n, y \in \mathbb{R}^m$ are the decision variablse, $f : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable and convex objective function, $\mathbb{K} \subset \mathbb{R}^m$ is a closed convex cone and $\mathbb{D} \subset \mathbb{R}^n$ is a closed convex set, $H \in \mathbb{R}^{m \times n}$ and $g \in \mathbb{R}^m$ are constraint parameters.

Analysis of Numerical Optimization Software

PIPG convergence result - essentials

Primal dual gap (optimality)

- Convergence with an order of $O(\frac{1}{k^2})$ to the optimum, in a primal-dual sense
- $L(\bar{z}, w^\star) - L(z^\star, \bar{w})$ is known as the *primal-dual gap* evaluated at $(\bar{z}, \bar{w})$
- Thus, (6) provides an upper bound for the primal dual gap of the iterate pair $(\bar{z}^k, \bar{w}^k)$.

Feasibility

- constraint violation is brought to zero with an order of $O(\frac{1}{k^3})$

Convergence results are shown for the $\bar{z}^k$ and $(\bar{z}^k, \bar{w}^k)$, respectively.

- convex combinations of all iterates up to index k with strictly increasing convex combination factors.

Ongoing development

Proportional Integral Projected Gradient

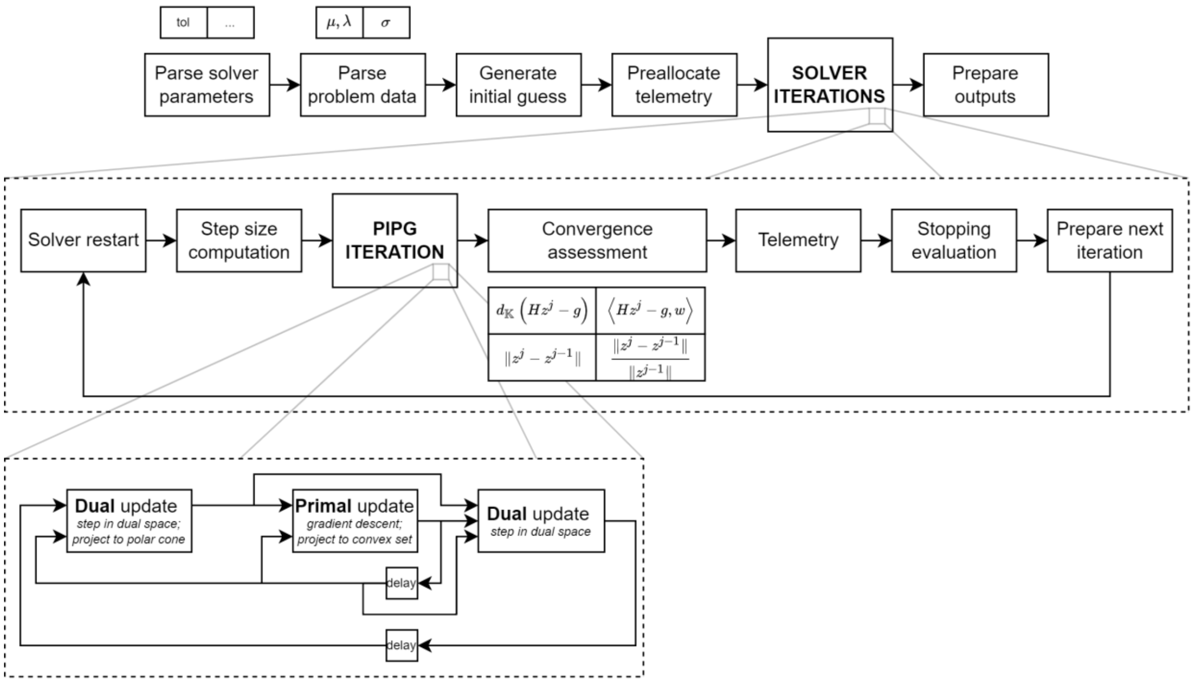
Implementation

General purpose implementation

$$\begin{aligned} \min \quad & \frac{1}{2}\mathbf{z}^T\mathbf{P}\mathbf{z} + \mathbf{c}^T\mathbf{z} & \\ \text{s.t.} \quad & \mathbf{z} \in \mathrm{D} & \text{closed convex set} \\ & \mathbf{H}\mathbf{z} - \mathbf{g} \in \mathrm{K} & \text{closed convex cone} \end{aligned}$$

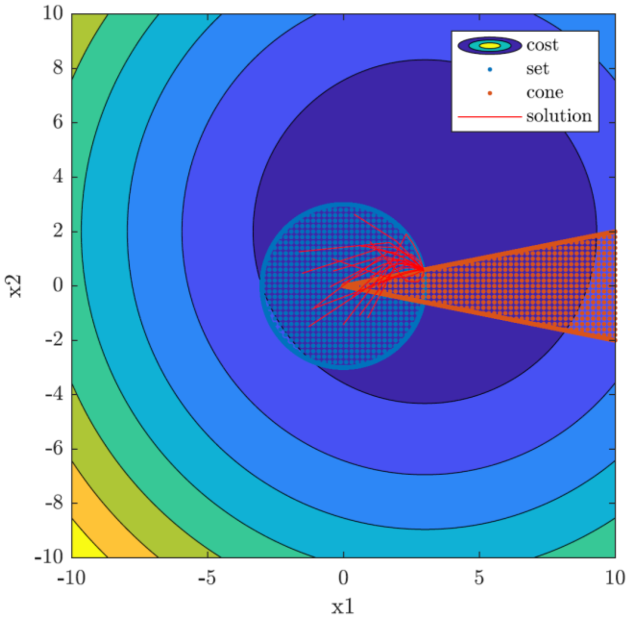
where

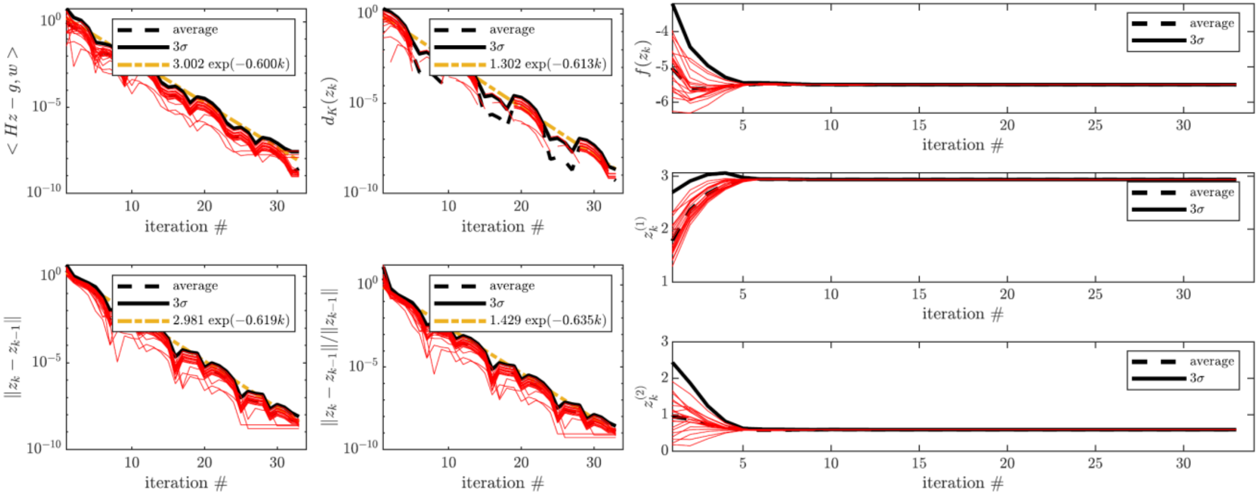
- $\mathrm{D} = \mathrm{D}_0 \times \cdots \times \mathrm{D}_n$ can be the cartesian product of *n* subsets
- $\mathrm{K} = \mathrm{K}_0 \times \cdots \times \mathrm{K}_m$ can be the cartesian product of *m* subcones



Proportional Integral Projected Gradient

Analysis

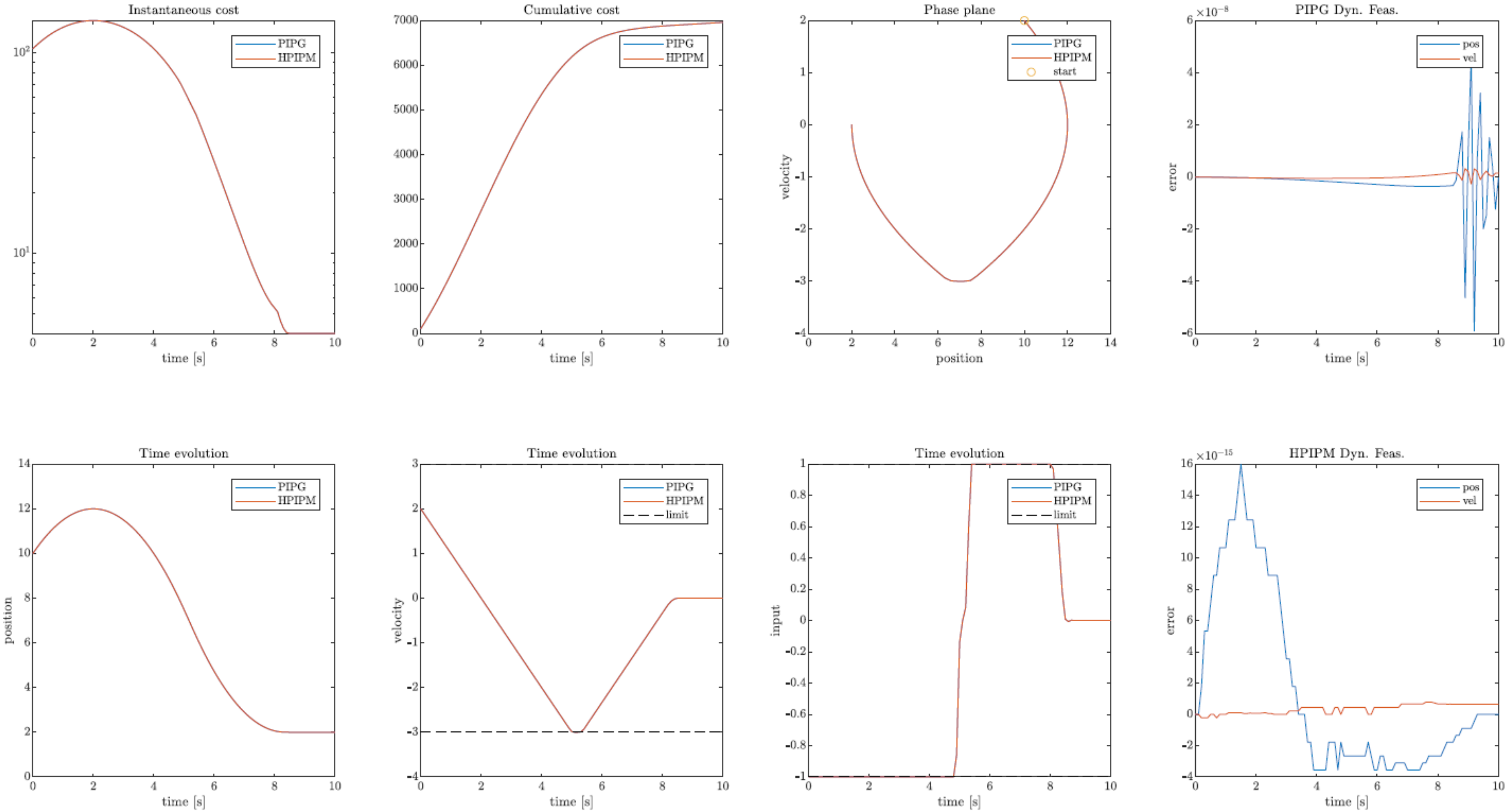




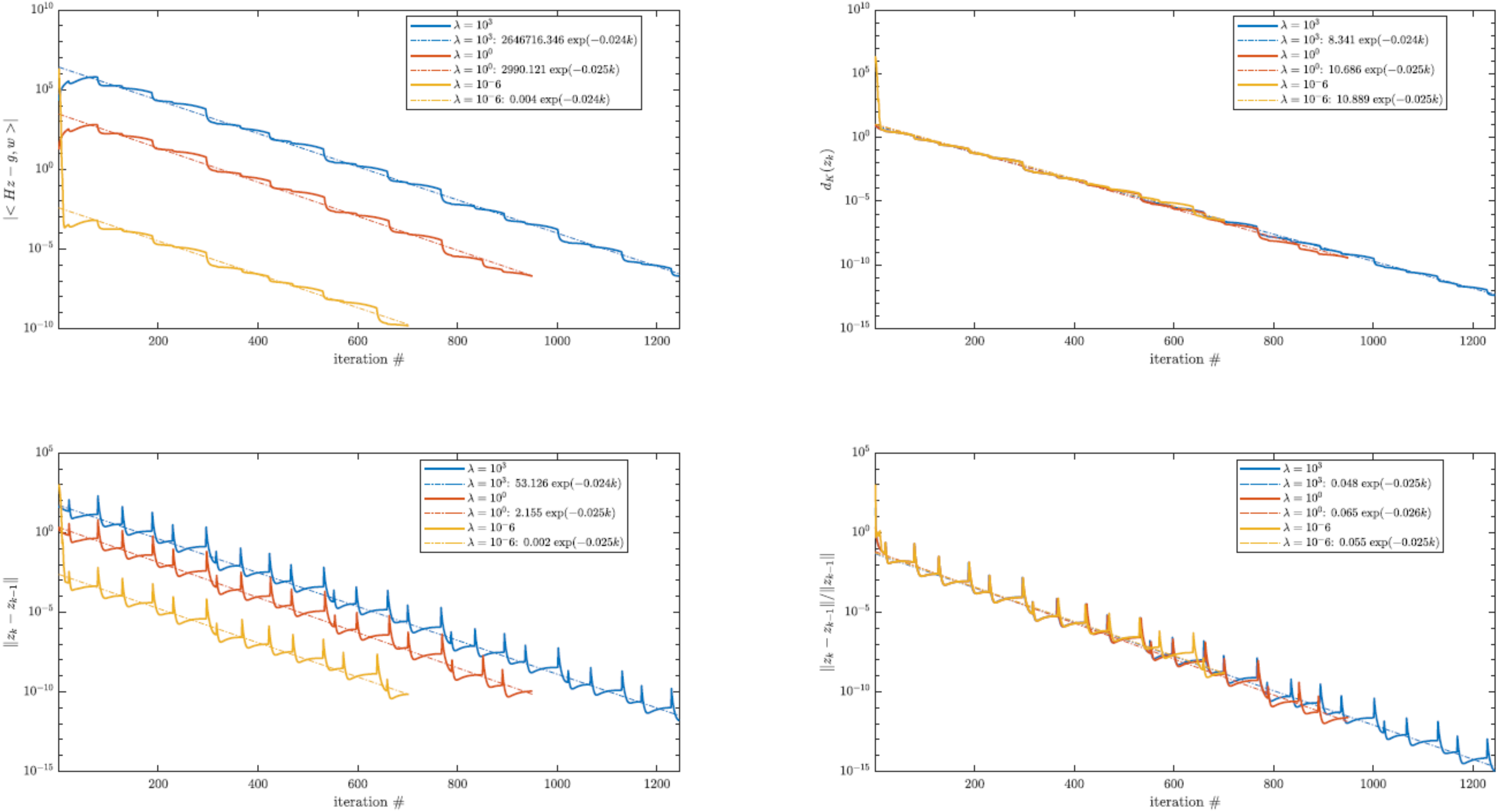
Proportional Integral Projected Gradient

Optimal Control

$$
\begin{aligned}
\underset{\substack{\mathbf{x}_1,\ldots,\mathbf{x}_N \\ \mathbf{u}_0,\ldots,\mathbf{u}_{N-1} \\ \mathbf{s}}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{s}^T\mathbf{M}\mathbf{s} + \mathbf{m}^T\mathbf{s} + \sum_{k=0}^{N-1}\Bigg(\frac{1}{2}\mathbf{x}_{k+1}^T\mathbf{Q}_{k+1}\mathbf{x}_{k+1} + \frac{1}{2}\mathbf{u}_k^T\mathbf{R}_k\mathbf{u}_k \\
& \qquad + \mathbf{x}_{k+1}^T\mathbf{N}_k\mathbf{u}_k + \mathbf{q}_{k+1}^T\mathbf{x}_{k+1} + \mathbf{r}_k^T\mathbf{u}_k\Bigg) \\
\text{subject to} \quad & \mathbf{\Phi}_k\mathbf{x}_k + \mathbf{\Gamma}_k\mathbf{u}_k + \mathbf{c}_k = \mathbf{x}_{k+1}, \quad \text{(Dynamics)}, \quad k = 0,\ldots,N-1, \\
& \mathbf{x}_{k+1} \in \mathbb{X}_{k+1}, \quad \text{(State)}, \quad k = 0,\ldots,N-1, \\
& \mathbf{u}_k \in \mathbb{U}_k, \quad \text{(Inputs)}, \quad k = 0,\ldots,N-1, \\
& \mathbf{s} \in \mathbb{S}, \quad \text{(Slacks)}, \\
& \mathbf{H}_k^{(0)}\mathbf{x}_k + \mathbf{H}_k^{(1)}\mathbf{x}_{k+1} + \mathbf{H}_k^{(u)}\mathbf{u}_k + \mathbf{H}_k^{(s)}\mathbf{s} - \mathbf{g}_k \in \mathbb{K}_k, \quad \text{(General)}, \quad k = 0,\ldots,N-1
\end{aligned}
$$



Instantaneous cost
PIPG
HPIPM
time [s]
Cumulative cost
PIPG
HPIPM
time [s]
Phase plane
PIPG
HPIPM
start
velocity
position
PIPG Dyn. Feas.
pos
vel
error
time [s]
Time evolution
PIPG
HPIPM
position
time [s]
Time evolution
PIPG
HPIPM
limit
velocity
time [s]
Time evolution
PIPG
HPIPM
limit
input
time [s]
HPIPM Dyn. Feas.
pos
vel
error
time [s]

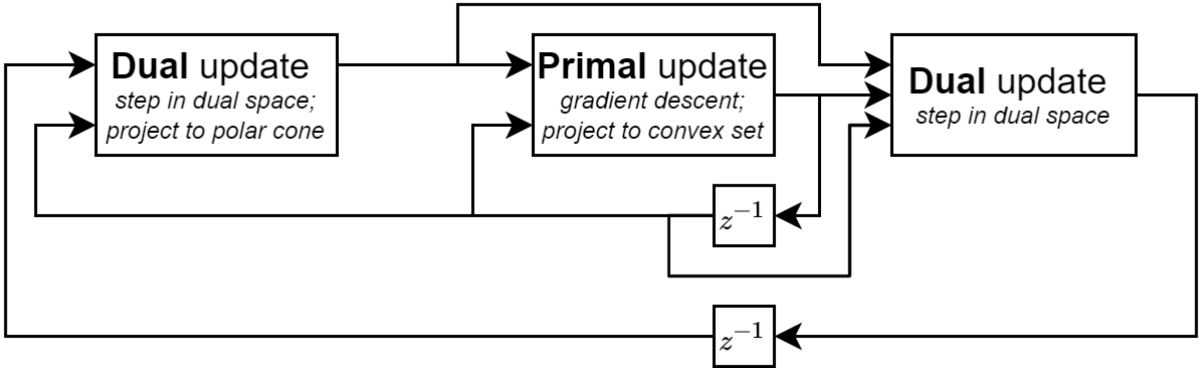


$\lambda = 10^3$
$\lambda = 10^3$: 2646716.346 exp(−0.024k)
$\lambda = 10^0$
$\lambda = 10^0$: 2990.121 exp(−0.025k)
$\lambda = 10^-6$
$\lambda = 10^-6$: 0.004 exp(−0.024k)
$| < Hz - g, w > |$
iteration #
$\lambda = 10^3$: 8.341 exp(−0.024k)
$\lambda = 10^0$: 10.686 exp(−0.025k)
$\lambda = 10^-6$: 10.889 exp(−0.025k)
$d_K(z_k)$
iteration #
$\lambda = 10^3$: 53.126 exp(−0.024k)
$\lambda = 10^0$: 2.155 exp(−0.025k)
$\lambda = 10^-6$: 0.002 exp(−0.025k)
$\|z_k - z_{k-1}\|$
iteration #
$\lambda = 10^3$: 0.048 exp(−0.025k)
$\lambda = 10^0$: 0.065 exp(−0.026k)
$\lambda = 10^-6$: 0.055 exp(−0.025k)
$\|z_k - z_{k-1}\| / \|z_{k-1}\|$
iteration #

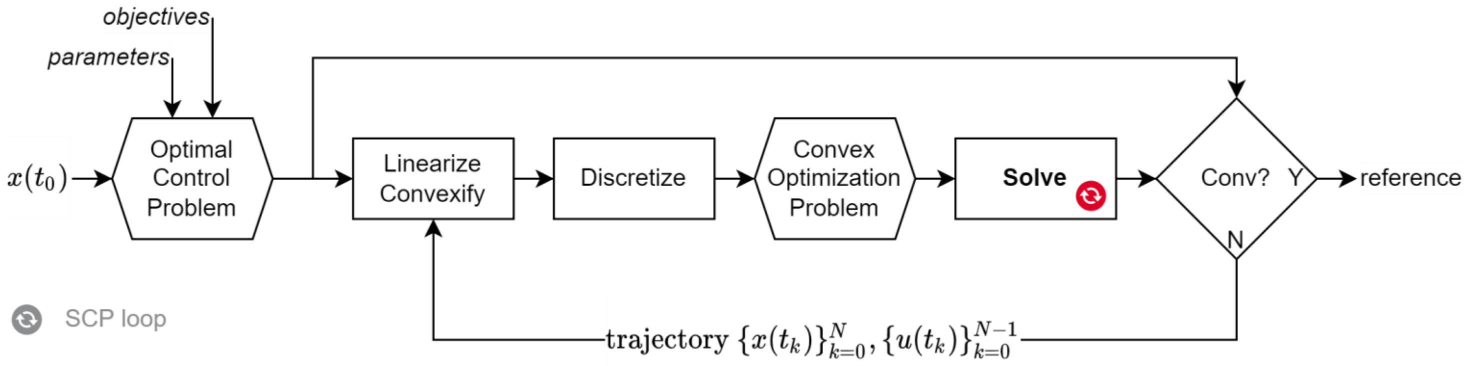
VV4RTOS

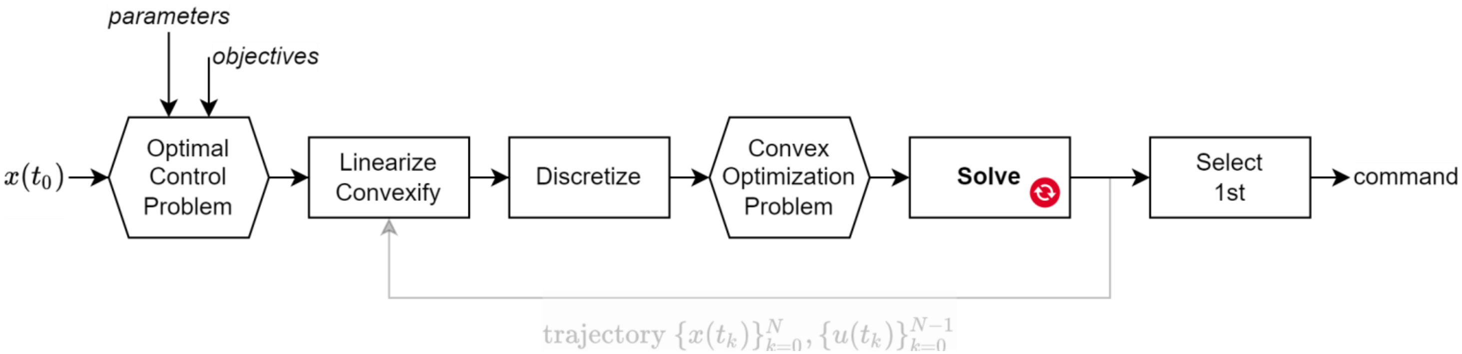
Where to now?

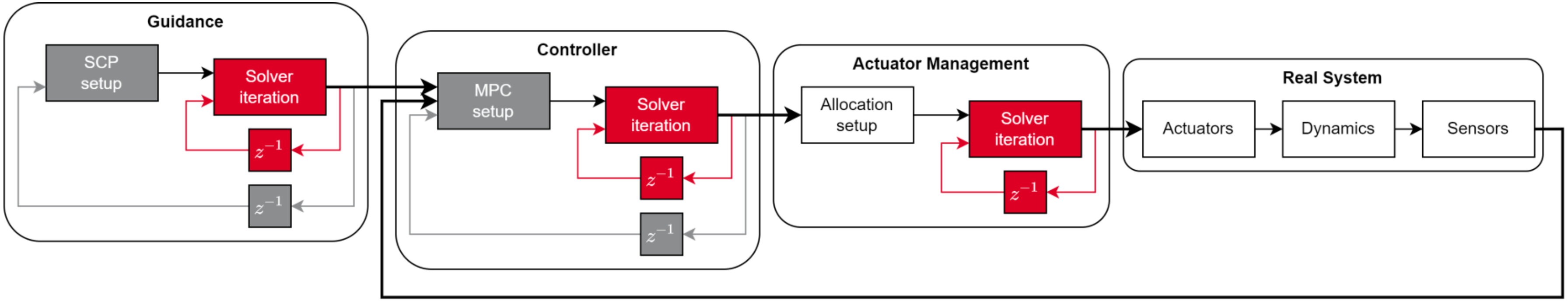
Solver loop

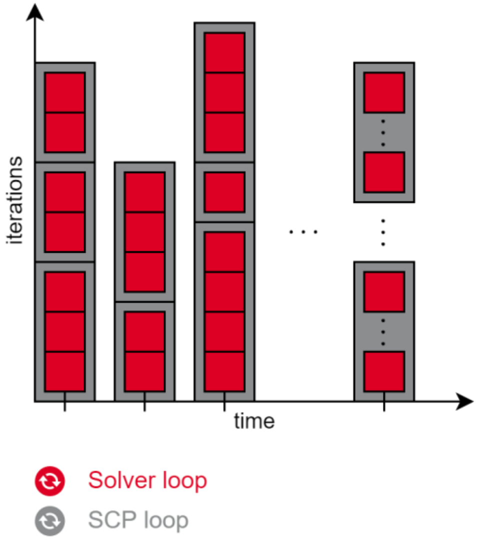


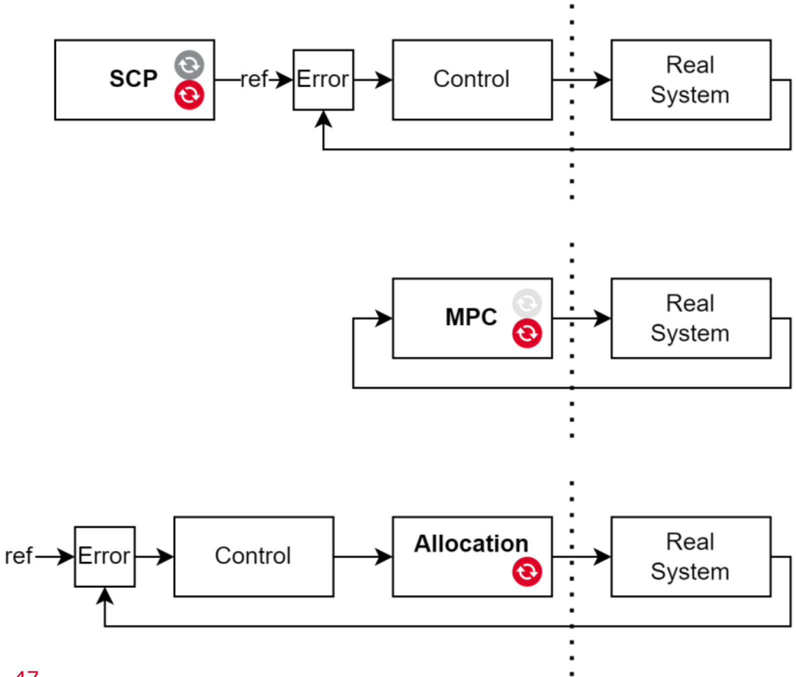
VV4RTOS


objectives
parameters
$x(t_0)$
Optimal Control Problem
Linearize Convexify
Discretize
Convex Optimization Problem
Solve
Conv?
Y
N
reference
SCP loop
trajectory $\{x(t_k)\}_{k=0}^{N}, \{u(t_k)\}_{k=0}^{N-1}$


parameters
objectives
$x(t_0)$
Optimal Control Problem
Linearize Convexify
Discretize
Convex Optimization Problem
Solve
Select 1st
command
trajectory $\{x(t_k)\}_{k=0}^{N}, \{u(t_k)\}_{k=0}^{N-1}$


Guidance
SCP setup
Solver iteration
z^{-1}
z^{-1}
Controller
MPC setup
Solver iteration
z^{-1}
z^{-1}
Actuator Management
Allocation setup
Solver iteration
z^{-1}
Real System
Actuators
Dynamics
Sensors


iterations
time
Solver loop
SCP loop


SCP
ref
Error
Control
Real System
MPC
Real System
ref
Error
Control
Allocation
Real System

Team Work + Passion for Challenges + Imagination + Innovation + Technology + Customer Focus + Hard Work

Thank you

Pedro Lourenço, on behalf of the VV4RTOS team
palourenco@gmv.com

INNOVATING SOLUTIONS